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MARKETS AND EFFICIENCY

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Intervention improves on competitive outcomes.
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1. Introduction

In 2001, Steven Weinberg[1] addressed the distinction between explanation and (mere) description in science: "If I had to give an a priori definition of explanation in physics I would say, "explanation in physics is what physicists have done when they say Aha! But a priori definitions (including this one) are not much use". Accomplished colleagues may not share this view, but academic economists have been absent from the recent and ongoing public debate concerning either the debt crisis in the eurozone or the financial crisis that set off the great recession; when not, their contributions have failed the aha criterion.

What to make of this failure? One way is to abandon what Frank Hahn called "grammatical thinking" in economics and the fascinating body of knowledge it has produced starting with the foundations of game theory and of the theory of competitive equilibrium more than half a century ago. Evidently, this will not do. Time and again rationality and equilibrium, rational expectations in particular, are held responsible for the failure of analytical macroeconomics to foresee and forestal the financial crisis and the recession. However, what is the rational expectations hypothesis other than simply grammatical thinking? Syntactical discipline imposed, if, formally, on the agents in a model, in essence, on the modeller?

Economics, and economic theory in particular, as taught in doctoral programs, provides tools and insights with implications for economic policy that are both immediate and far reaching. Tools that can prevent adverse developments in the allocation of resources or, if prevention is too much to ask for, at least intuitions that shed light on events, provide hints about possible ways out, and allow policy-makers to better practical experience and common sense.

There are two general situations in which markets fail and intervention is called for: economies with external effects or strategic interactions and economies with an incomplete asset market; together, they encompass economies with asymmetric information, moral hazard and adverse selection. Indeed, earlier, Prescott and Townsend (1984a,b) and, more recently, Minelli and Polemarchakis (2000), Dubey and Geanakoplos (2002), Allen and Gale (2004), Prescott and Townsend (2006) and Bisin *et al*. (2011) showed that adverse selection and moral hazard[2] can be recast in a more standard general equilibrium context.

Formally, *markets fail* or, equivalently, competitive allocations are constrained suboptimal if there exist Pareto improving interventions compatible with constraints that prevail, in particular, the constraints under which markets operate.

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1 See Weinberg (2001).

2 Benchmark formulations are Akerlof (1970), Rothschild and Stiglitz (1976) and Mirrlees (1999).

In Geanakoplos and Polemarchakis (2008), with operative external effects, there is a way to make everybody better off than they would be under perfect competition: by taxing or subsidising commodities anonymously (everyone pays the same tax) and redistributing the tax revenue anonymously. An alternative approach would be to ask which allocations can be implemented as strategic equilibria, through the design of mechanisms and an explicit recognition of incentive compatibility constraints;[3] but, it does not focus, explicitly at least, on competitive, anonymous markets.[4]

The asset market is complete if all contracts for the transfer of revenue over time and across realizations of uncertainty are priced and traded, and all individuals can participate in this market for assets with no restrictions; otherwise it is incomplete. With an incomplete asset market, constrained (sub)optimality, defined by Diamond (1967), is formally shown in Hart (1975); in Geanakoplos and Polemarchakis (1986), it is shown to be robust or generic. In Carvajal and Polemarchakis (2011), the argument is extended to economies subject to aggregate as well as uninsurable idiosyncratic risk, even if the asset market for the allocation of aggregate risks is complete.

Recent attempts to provide analytical foundations for Keynesian phenomena, most notably deficient aggregate demand and persistent unemployment, such as Chamley (2014), without recourse to ad hoc price rigidities or constraints on endogenous variables, exploit precisely the strategic interaction of agents.

The demonstration that in empirically compelling situations competitive allocations are constrained suboptimal makes an important methodological point. Departures from laissez-faire are often said to be counterproductive because competitive equilibrium cannot be Pareto improved upon. Because constraints on the operation of perfect competitive markets (such as externalities, strategic interactions and incomplete asset markets) are ubiquitous, such a view is untenable.

2. Pareto improving taxes[5]

Individuals are $i = 1, 2$ and commodities are $l = 1, 2, 3$.

The utility function of an individual is

$$u^i(x^i, x^{-i}) = u^{*i}(x^i) + \lambda^i_{-i,1} x_1^{-i} + \lambda^i_{-i,2} x_2^{-i} + \lambda^i_{-i,3} x_3^{-i},$$

where $\lambda^i = (\lambda^i_{-i,1}, \lambda^i_{-i,2}, \lambda^i_{-i,3})$ are the coefficients of external effects and

$$u^{*i}(x^i) = x_1^i - \frac{1}{2}\alpha_1^i (x_1^i)^2 + x_2^i - \frac{1}{2}\alpha_2^i (x_2^i)^2 + x_3^i, \quad 0 < \alpha_1^i, \alpha_2^i < 1$$

is the private utility function over own consumption. Externalities are separable. The endowments of individuals are

$$e^1 = (1, 0, e_3^1) \quad \text{and} \quad e^2 = (0, 1, e_3^2),$$

respectively, with the endowment in commodity $l = 3$ sufficiently large.

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3 Hurwicz (1972, 1979) and Maskin (1999) provide foundations for the theory of implementation.

4 Ueda (2013) argues that intermediation can promote efficiency in economies with production externalities.

5 See Geanakoplos and Polemarchakis (2008).

Prices of commodities are $p = (p_1, p_2, 1)$, and tax rates on commodities are $t = (t_1, t_2, 0)$; commodity $l = 3$ is the numéraire and not subject to taxation. External effects that are separable from the marginal utility of each individual's own consumption allow for competitive equilibrium prices and allocations, with or without taxes, that are independent of the coefficients of external effects. The quasi-linearity of the utility functions in the numéraire commodity eliminates income effects and facilitates computations. Competitive equilibrium prices are

$$p_1(t) = 1 - \frac{1}{\frac{1}{\alpha_1^1} + \frac{1}{\alpha_1^2}}\left(1 + \frac{1}{\alpha_1^2} t_1\right),$$

$$p_2(t) = 1 - \frac{1}{\frac{1}{\alpha_2^1} + \frac{1}{\alpha_2^2}}\left(1 + \frac{1}{\alpha_2^1} t_2\right),$$

and equilibrium allocations are

$$x_1^1(t) = \frac{1}{\alpha_1^1 + \alpha_1^2}(\alpha_1^2 + t_1),$$

$$x_2^1(t) = \frac{1}{\alpha_2^1 + \alpha_2^2}(\alpha_2^2 - t_2),$$

$$x_3^1(t) = e_3^1 + p_1(t)x_1^2(t) - (p_2(t) + t_2)x_2^1(t) + \frac{1}{2}(t_1 x_1^2(t) + t_2 x_2^1(t)),$$

$$x_1^2(t) = \frac{1}{\alpha_1^1 + \alpha_1^2}(\alpha_1^1 - t_1),$$

$$x_2^2(t) = \frac{1}{\alpha_2^1 + \alpha_2^2}(\alpha_2^1 + t_2),$$

$$x_3^2(t) = e_3^2 + p_2(t)x_2^1(t) - (p_1(t) + t_1)x_1^2(t) + \frac{1}{2}(t_1 x_1^2(t) + t_2 x_2^1(t)).$$

The derivative of utilities with respect to taxes at the point of zero taxes is obtained first by computing the effect on consumption and prices:

$$\frac{dp_1}{dt_1} = -\frac{1}{\frac{1}{\alpha_1^1} + \frac{1}{\alpha_1^2}}\frac{1}{\alpha_1^2}$$

$$\frac{dx_2^1}{dt_1} = \frac{dx_2^2}{dt_1} = 0$$

$$\frac{dx_1^2}{dt_1} = -\frac{dx_1^1}{dt_1} = -\frac{1}{a_1^1 + a_1^2}$$

$$e_1^1 - x_1^1 = \frac{a_1^1}{a_1^1 + a_1^2}$$

$$\frac{d\tau}{dt_1} = \frac{1}{2} x_1^2 = \frac{1}{2} \frac{a_1^1}{a_1^1 + a_1^2}$$

$$\frac{dx_3^1}{dt_1} = (e_1^1 - x_1^1)\frac{dp_1}{dt_1} + \frac{d\tau}{dt_1} = \frac{\alpha_1^1(\alpha_1^2 - \alpha_1^1)}{2(\alpha_1^1 + \alpha_1^2)^2}$$

$$\frac{dx_3^2}{dt_1} = -x_1^2\left(\frac{dp_1}{dt_1} + 1\right) + \frac{d\tau}{dt_1} = -\frac{\alpha_1^1(\alpha_1^2 - \alpha_1^1)}{2(\alpha_1^1 + \alpha_1^2)^2}.$$

By the envelope theorem and the fact that the marginal utility of consumption is 1, the change in utility from consumption to agent 1 is just $\frac{dx_3^1}{dt_1}$. The effect on his or her total utility also includes the externalities from the changes in 2’s consumption; namely,

$$\begin{aligned}\frac{du_1^1}{dt_1} &= \frac{dx_3^1}{dt_1} + \frac{dx_1^2}{dt_1}\lambda_{2,1}^1 + \frac{dx_2^2}{dt_1}\lambda_{2,2}^1 + \frac{dx_3^2}{dt_1}\lambda_{2,3}^1 \\ &= \frac{\alpha_1^1(\alpha_1^2 - \alpha_1^1)}{2(\alpha_1^1 + \alpha_1^2)^2} + \frac{-1}{\alpha_1^1 + \alpha_1^2}\lambda_{2,1}^1 + 0 - \frac{\alpha_1^1(\alpha_1^2 - \alpha_1^1)}{2(\alpha_1^1 + \alpha_1^2)^2}\lambda_{2,3}^1.\end{aligned}$$

By symmetry, the other welfare effects can be immediately deduced to be

$$D_t\vec{u} = \begin{pmatrix} D_t u^1 \\ D_t u^2 \end{pmatrix} = \begin{pmatrix} \frac{\partial u^1}{\partial t_1} & \frac{\partial u^1}{\partial t_2} \\ \frac{\partial u^2}{\partial t_1} & \frac{\partial u^2}{\partial t_2} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{\alpha_1^1(\alpha_1^2 - \alpha_1^1)}{2(\alpha_1^1 + \alpha_1^2)^2}(1 - \lambda_{2,3}^1) - \frac{1}{\alpha_1^1 + \alpha_1^2}\lambda_{2,1}^1 & \frac{\alpha_2^2(\alpha_2^2 - \alpha_2^1)}{2(\alpha_2^1 + \alpha_2^2)^2}(1 - \lambda_{2,3}^1) + \frac{1}{\alpha_2^1 + \alpha_2^2}\lambda_{2,2}^1 \\ \frac{\alpha_1^1(\alpha_1^1 - \alpha_1^2)}{2(\alpha_1^1 + \alpha_1^2)^2}(1 - \lambda_{1,3}^2) + \frac{1}{\alpha_1^1 + \alpha_1^2}\lambda_{1,1}^2 & \frac{\alpha_2^2(\alpha_2^1 - \alpha_2^2)}{2(\alpha_2^1 + \alpha_2^2)^2}(1 - \lambda_{1,3}^2) - \frac{1}{\alpha_2^1 + \alpha_2^2}\lambda_{1,2}^2 \end{pmatrix}.$$

In the absence of external effects, for $\vec{\lambda} = 0$, the matrix $D_t\vec{u}$ is singular; in particular, $D_t u^1 + D_t u^2 = 0$, and Pareto improving taxes do not exist: the Walrasian equilibrium is Pareto optimal and (normalized) changes in utility sum to 0 across individuals.

Pareto improving taxes, dt, that solve $(D_t\vec{u})dt \gg 0$, exist if the matrix $D_t\vec{u}$ has full row rank.

Because the coefficients $\frac{1}{\alpha_\ell^i + \alpha_\ell^{-i}}$ are all non-zero (in fact, all positive), it is clear that by perturbing the variables $\lambda_{-i,l}^i$ one can perturb the matrix in any way desired. Thus, the matrix $D_t\vec{u}$ is invertible for almost all choices of the externality variables $\lambda_{-i,l}^i$. Because a regular economy $\vec{e}$ has a finite number of equilibria, almost all choices $\lambda^{i}_{-i,l}$ will simultaneously make all the equilibrium welfare effect matrices $D_t\vec{u}$ invertible.

3. Public investment[6]

Activity extends over 2 days, 0 and 1, and there is only a continuum of ex-ante identical individuals. One commodity is available for exchange and consumption in date 0, and it is storable. At date 1, there are two commodities: individuals trade and consume what they have stored of the first commodity, along with any further endowment they may have, and they also exchange and consume a second commodity. The amount of commodity that is stored by each individual at date 0 is k, and e_2 is the endowment of commodity 2 they all receive at date 1. The endowment of commodity 1 is subject to idiosyncratic risk: for an individual in personal state s, the endowment of commodity 1 is $e_{1,s}$. The proportion of individuals in state s is π_s, and $e_1 = \mathbb{E}_\pi[e_{1,s}]$. With Bernoulli utility indices v_s, the individual's von Neumann-Morgenstern ex-ante utility is

$$u(k, x) = -k + \mathbb{E}_\pi[v_s(x_s)],$$

where $x_s = (x_{1,s}, x_{2,s})$ is the individuals' consumption at date 1 in state s; quasi-linearity facilitates computations. Commodity 1 is the numéraire at date 1, and prices are by $(1, p)$; the wealth of an individual in personal state s is $e_{1,s} + k + pe_2$.

If λ_s is the marginal utility of income in personal state s, the first-order condition for optimization at date 0 is that $\mathbb{E}_\pi[\lambda_s] = 1$, and, as a consequence, the ex-ante utility impact of an infinitesimal perturbation to the level of savings, dk, around its competitive equilibrium level, is

$$du = -dk + \mathbb{E}_\pi\{\lambda_s[(-x_{2,s} + e_2)dp + dk]\} = \mathbb{E}_\pi[\lambda_s(-x_{2,s} + e_2)]dp.$$

Market clearing requires that

$$\mathbb{E}_\pi(-x_{2,s} + e_2) = 0.$$

Under certainty, it is impossible to improve ex-ante utility through the implementation of levels of savings different from the ones chosen under competitive equilibrium. Under uncertainty, the equilibrium utility level can be improved upon if

$$\mathbb{E}_\pi[\lambda_s(-x_{2,s} + e_2)] \neq 0.$$

A specific structure allows for a closed-form solution. The Bernoulli indices are state-independent and

$$v(x_s) = \ln x_{1,s} + \ln x_{2,s}.$$

By direct computation, given savings of k, equilibrium prices are $p = (e_1 + k)/e_2$, an expression that depends positively on k. Also, $\lambda_s = 2/(e_{1,s} + e_1 + 2k)$ and

[6] See Carvajal and Polemarchakis (2011).

$$e_2 - x_{2,s} = \frac{e_2}{2}\left(\frac{e_1 - e_{1,s}}{e_1 + k}\right).$$

If, furthermore, there are only two personal states, $e_{1,s} = e_1 \pm \varepsilon$, that occur with equal probability, then

$$\mathbb{E}_\pi[\lambda_s(e_2 - x_{2,s})] = \frac{1}{e_1 + k}\left(\frac{\varepsilon^2}{4(e_1 + k)^2 - \varepsilon^2}\right).$$

At the equilibrium level of savings, $\mathbb{E}_\pi[\lambda_s] = 1$. By direct computation,

$$\mathbb{E}_\pi[\lambda_s(e_s - x_{2,s})] = \frac{\varepsilon^2}{4(e_1 + k)^2} > 0.$$

At a competitive equilibrium, individuals underinvest.

A variation allows for production; as with simple storage, a two-period economy is populated by a continuum of mass 1 of identical individuals. Again, a single commodity is available in the first period, and it can be either consumed or invested: the amount of this commodity that is saved, k, becomes the endowment for the second period. In the second period, this level k of capital is combined with a second production factor, labour, to produce a consumption good; the amount of labour used is l and c is the consumption good produced. In the first period, individuals only consume and save. In the second period, they are endowed with $\bar{l}$ units of labour that they supply inelastically, together with their savings k, in exchange for the consumption good. Initially, there is no risk, so that all the individuals are endowed with $\bar{l}$ at date 1. The overall utility of the individuals is given by

$$u(k, c) = -k + v(c),$$

where v is the person's utility index for consumption in the second period. The consumption good is the numèraire of the second period, and denoting the price of capital as $1 + r$ and the price of labour as w, the budget date-1 wealth of the individuals is

$$\tau(k) = (1 + r)k + w\bar{l},$$

which they use to buy consumption, so $c = \tau(k)$.

The technology of production is

$$y = f(k, l) + (1 - \delta)k,$$

where function f is assumed to exhibit constant returns to scale. It is immediate, then, from the maximization of profits, that $(1 + r) = f_k + (1 - \delta)$ and $w = f_l$, while[7]

[7] In these expressions and henceforth, we omit the arguments, $(k, \bar{l})$, of function f, for the sake of notational simplicity.

$$\frac{\partial r}{\partial k} = f_{kk}$$

and

$$\frac{\partial w}{\partial k} = f_{lk} = f_{kl}.$$

From the optimization of the individuals, it must be that

$$\lambda = \frac{1}{1+r},$$

where λ represents the marginal utility of consumption at date 1. The latter implies that[8]

$$du = -dk + \lambda(kdr + \bar{l}dw + (1+r)dk) = \frac{kdr + \bar{l}dw}{1+r} = \frac{kf_{kk} + \bar{l}f_{kl}}{1+r}dk,$$

which, because f_k is homogeneous of degree 0, is $du = 0$, an equality that confirms that, under certainty, the privately determined level of investment cannot be improved upon.

Now, suppose that the endowment of labour in the second period is subject to idiosyncratic risk, and is $\bar{l}_s = \bar{l} + \varepsilon_s$ in personal state s, which occurs with probability π_s. As before, we can let

$$\tau_s(k) = (1+r)k + w\bar{l}_s$$

be the individuals' wealth in personal state s.

With ex-ante preferences

$$u(k, c) = -k + \mathbb{E}_\pi[v_s(c_s)],$$

if we let λ_s be the marginal utility of revenue in state s, we have that the first-order condition of the individuals at date 0 is

$$\mathbb{E}_\pi[\lambda_s] = \frac{1}{1+r}.$$

As a consequence,

$$du = \mathbb{E}_\pi[\lambda_s(kf_{kk} + \bar{l}_s f_{kl})]dk,$$

which becomes simply

[8] In this case, because there is only one commodity at date 1, the computation of λ is immediate. In the case of multiple commodities, one would express the indirect utility of the individual as $V(\tau) = v(c(p, \tau))$ and use $\lambda = V'$. In general, also, with a consumption plan c, endowment e and prices p, one would have $du = -dk + \lambda(-cdp + edp + (1+r)dk)$.

$$du = \mathbb{E}_\pi[\lambda_s \varepsilon_s] f_{kl} dk:$$

the equilibrium allocation is constrained suboptimal, as long as $\mathbb{E}_\pi[\lambda_s \varepsilon_s] \neq 0$.

Once again, for a closed-form solution we can consider a particular functional form for the Bernoulli indices, letting

$$v_s(c_s) = \frac{\beta}{\gamma} c_s^\gamma,$$

where $\beta > 0$ and $\gamma < 1$. In this case, the marginal utilities of income are given by

$$\lambda_s = \beta c_s^{\gamma-1} = \beta(y + \varepsilon_s f_l)^{\gamma-1},$$

so the first-order condition of the individuals' optimization problem is

$$\frac{1}{1+r} = \beta \mathbb{E}_\pi[(y + \varepsilon_s f_l)^{\gamma-1}]$$

while

$$du = \beta \mathbb{E}_\pi[(y + \varepsilon_s f_l)^{\gamma-1} \varepsilon_s] f_{kl} dk.$$

If we further assume, as before, two equally probable personal states, with $\varepsilon_s = \pm\varepsilon$, then the first-order condition becomes

$$\frac{1}{2}\beta(y + \varepsilon f_l)^{\gamma-1} + \frac{1}{2}\beta(y - \varepsilon f_l)^{\gamma-1} = \frac{1}{1+r},$$

which implies, because $\varepsilon > 0$ and $\gamma < 1$, that

$$\frac{1}{2}\beta(y - \varepsilon f_l)^{\gamma-1} > \frac{1}{1+r}.$$

Because in this case, by substitution,

$$\mathbb{E}_\pi[\lambda_s \varepsilon_s] = \frac{1}{2}\beta(y + \varepsilon f_l)^{\gamma-1}(\varepsilon) + \frac{1}{2}\beta(y - \varepsilon f_l)^{\gamma-1}(-\varepsilon) = \beta\varepsilon(1 - (y - \varepsilon f_l)^{\gamma-1}) < 0,$$

and because $f_{kl} > 0$, it follows that individuals overinvest at date 0: if $dk < 0$, then $du > 0$.

4. Public debt[9]

The economy is as in Diamond (1967), where each generation lives for two periods, and the population grows at a constant rate $n \geq 0$.

[9] See Carvajal and Polemarchakis (2011).

Initially, the individuals of the economy are endowed with l units of labour *in each of the two periods they live*, which they supply inelastically. Using the same notation as in the previous example, their ex-ante utility is given by

$$u(k) = \bar{l}w - k + v(\bar{l}w + (1+r)k),$$

and the effects of a perturbation are

$$du = \bar{l}dw - dk + \lambda(kdr + \bar{l}dw + (1+r)dk).$$

In this case, the first-order conditions of individual optimization are that

$$\lambda = \frac{1}{1+r},$$

and, by direct substitution,

$$du = \frac{(2+r)\bar{l}dw + kdr}{1+r}.$$

Here, because the total supply labour is $(2+n)\bar{l}$, we get, from the fact that the production technology is of constant returns to scale, that

$$(2+n)\bar{l}dw + kdr = 0,$$

so

$$du = \frac{r-n}{1+r}\bar{l}f_{lk}dk,$$

which establishes the *Golden Rule* criterion: if the interest rate is above (respectively, below) the rate of population growth, in equilibrium the economy underinvests (respectively, overinvests).

Now, the endowment of labour in the second period is subject to idiosyncratic risk, and is $\bar{l}_s = \bar{l} + \varepsilon_s$ with probability π_s, where $\mathbb{E}_\pi[\varepsilon_s] = 0$. As before, letting λ_s be the marginal utility of income in state s, we have that the first-order condition for optimization is

$$\mathbb{E}_\pi[\lambda_s] = \frac{1}{1+r},$$

and, hence, the welfare effects of a perturbation are

$$du = \bar{l}dw - dk + \mathbb{E}_\pi[\lambda_s(kdr + \bar{l}_s dw + (1+r)dk)] = \left(\frac{r-n}{1+r}\bar{l} + \mathbb{E}_\pi[\lambda_s\varepsilon_s]\right)f_{lk}dk.$$

If we assume that

$$v(c) = \frac{\beta}{\gamma} c^{\gamma},$$

as in the previous example, then

$$\lambda_s = \beta c_s^{\gamma-1} = \beta(y - (1+n)\bar{l}w + \varepsilon_s w)^{\gamma-1}.$$

If we further take that $\varepsilon_s = \pm\varepsilon$ with probability 1/2, then the first-order condition becomes

$$\mathbb{E}_{\pi}[\lambda_s] = \frac{1}{2}\beta(y - (1+n)\bar{l}w + \varepsilon w)^{\gamma-1} + \frac{1}{2}\beta(y - (1+n)\bar{l}w - \varepsilon w)^{\gamma-1} = \frac{1}{1+r},$$

which implies that

$$\frac{1}{2}\beta(y - (1+n)\bar{l}w - \varepsilon w)^{\gamma-1} > \frac{1}{1+r}.$$

By substitution,

$$\mathbb{E}_{\pi}[\lambda_s \varepsilon_s] = \frac{1}{2}\beta(y - (1+n)\bar{l}w + \varepsilon w)^{\gamma-1}\varepsilon - \frac{1}{2}\beta(y - (1+n)\bar{l}w - \varepsilon w)^{\gamma-1}\varepsilon$$
$$= \varepsilon\left(\frac{1}{1+r} - \beta(y - (1+n)\bar{l}w - \varepsilon w)^{\gamma-1}\right) < 0.$$

Recalling the expression for *du*, it follows that when the interest rate is below the rate of population growth the competitive equilibrium implies overinvestment (as in the case of certainty). However, now, in the presence of idiosyncratic risk, the second prescription of the Golden Rule *may* fail, and in an economy where the interest rate is higher than the growth of population, it may be that a Pareto improvement requires that every generation save less.

It remains an issue, evidently, whether policy-makers know enough about the population distribution of fundamentals to intervene, as in Geanakoplos and Polemarchakis (1990) and Kubler *et al*. (2002), or whether the lack of information about fundamentals available to policy-makers, "ignorance", supports the invisible hand. In Carvajal and Polemarchakis (2008), finite data sets of equilibrium information may be insufficient for the identification of Pareto-improving policies: Pareto-improving policies exist when market incompleteness allows the vectors of marginal utilities of revenue across states of the world of different individuals to diverge from co-linearity even at equilibrium. However, with vectors of marginal utilities of revenue that are neither identified nor collinear, Pareto-improving policies are unclear: on the basis of observed data, a profile of preferences in which the policy leaves at least one individual worse off cannot be ruled out.

5. Monetary policy: Quantitative easing[10]

The conclusion, that, surprisingly, had gone unnoticed, is that monetary policy determines the path of expected or average inflation, but not the distribution of possible paths of

[10] McMahon *et al*. (2012).

inflation. The stochastic path of inflation is determined by the manner in which monetary authorities adjust their portfolios over time. Under normal operations of monetary policy, the balance sheets of central banks are predominantly composed of short-term assets and held to conform with a target maturity or duration structure. Indeed, the fiscal theory of the price level in Woodford (1994) or Dubey and Geanakoplos (2003, 2006) takes for granted that monetary authorities trade exclusively in short-term nominally risk-free bonds, and it fails to highlight the importance of this assumption for the claim of determinacy of the path of prices. Bloise *et al*. (2005) and Nakajima and Polemarchakis (2005) indicate the importance of the assumption, but do not emphasize its policy implications; Drèze and Polemarchakis (2000), Adao *et al*. (2014) and Magill and Quinzii (2014b) point out that "comprehensive monetary policy", that sets the prices of contingent claims or the term structure of interest rates and not only the one-period rate can serve to guarantee determinacy, but they do not make the connection with credit easing; Magill and Quinzii (2014a) emphasize the role of inflationary expectations. However, under unconventional policies, the asset side of the portfolio is more varied and is often not ex ante specified, and it is dependent on market forces, and ultimately market expectations that determine the value of the assets held by the central bank. Variation in the value of the balance sheet of the central bank then determines the stochastic path in which money is injected or withdrawn, determining the path of inflation.

A representative individual derives utility from leisure and consumption according to the cardinal utility index

$$u(l, c) = \ln(\overline{l} - l) + \ln c,$$

where l is labour supplied, $\overline{l}$ is the endowment in leisure and c is consumption; and the intertemporal utility function of the individual is

$$u(l_0, c_0, \ldots, l_s, c_s, \ldots) = \ln(\overline{l}_0 - l_0) + \ln c_0 + \delta \sum_s \ln(\overline{l}_s - l_s) + \ln c_s.$$

Labour input produces output, consumption, with constant marginal cost of 1. The real wage is 1 and the price of consumption is p. Holdings of money balances are m, of one-period elementary securities f, and of one-period risk-free bonds b; evidently, elementary securities are traded only when uncertainty remains to be resolved. Prices of elementary securities are q, and the one-period nominal rate of interest is r.

The sequence of budget constraints is

$$p_0 c_0 + m_0 + \sum_s q_s f_s \leq p_0 l_0 + w,$$

$$p_s c_s + m_s + \frac{1}{1 + r_s} b_s \leq p_s l_s + m_0 + f_s,$$

$$0 \leq m_s + b_s,$$

and the cash-in-advance constraints are

$$p_0 l_0 \leq m_0,$$

$$p_s l_s \leq m_s.$$

Equivalently, the individual faces the cumulative budget constraint

$$p_0 c_0 + \sum_s \tilde{p}_s c_s \leq \frac{1}{1+r_0} p_0 l_0 + \sum_s \frac{1}{1+r_s} \tilde{p}_s l_s + t,$$

where

$$\tilde{p}_s = q_s p_s.$$

From the maximization of individual utility,

$$\begin{aligned} c_0 &= \frac{1}{2(1+\delta)} \frac{1}{p_0} \tau, \\ l_0 &= \overline{l}_0 - \frac{1}{2(1+\delta)} \frac{1+r_0}{p_0} \tau, \\ c_s &= \frac{\pi_s \delta}{2(1+\delta)} \frac{1}{\tilde{p}_s} \tau, \\ l_s &= \overline{l}_s - \frac{\pi_s \delta}{2(1+\delta)} \frac{1+r_s}{\tilde{p}_s} \tau, \end{aligned}$$

where

$$\tau(p_0, r_0, \ldots, p_s, r_s, \ldots) = \frac{1}{1+r_0} p_0 \overline{l}_0 + \sum_s \frac{1}{1+r_s} \tilde{p}_s \overline{l}_s + w.$$

Equilibrium in the consumption and labour markets requires that

$$\begin{aligned} c_0 &= l_0, \\ c_s &= l_s. \end{aligned}$$

It follows that

$$\begin{aligned} p_0 &= \frac{1}{\overline{l}_0}(2+r_0)\frac{1}{2(1+\delta)}\frac{1}{1-a} w, \\ \tilde{p}_s &= \frac{1}{\overline{l}_s}(2+r_s)\frac{\pi_s \delta}{2(1+\delta)}\frac{1}{1-a} w, \end{aligned}$$

where

$$a(r_0, \ldots, r_s, \ldots) = \frac{(2+r_0)}{(1+r_0)} \frac{1}{2(1+\delta)} + \sum_s \frac{(2+r_s)}{(1+r_0)} \frac{\pi_s \delta}{2(1+\delta)}$$

and

$$c_0 = l_0 = \frac{\overline{l}_0}{2 + r_0},$$
$$c_s = l_s = \frac{\overline{l}_s}{2 + r_s}.$$

Given w, that is, with a non-Ricardian specification, equilibrium in the goods markets determines the price level or, equivalently, p_0, but, it does not determine the decomposition of $\tilde{p}_s = q_s p_s$.

Determinacy obtains if money balances are issued against treasury bills, one-period nominally risk-free bonds: in this case,

$$f_s = f_{s'}, \quad all\ s, s'.$$

Because, at equilibrium,

$$\frac{r_s}{1 + r_s} p_s l_s = m_0 + f_s,$$

the requirement that open market operations be restricted to treasury bills, together with the non-arbitrage condition

$$\frac{1}{1 + r_0} = \sum_s q_s,$$

completes the determination of equilibrium prices; in particular,

$$q_s = \frac{\pi_s}{1 + r_0} \quad or \quad p_s = (1 + r_0)(2 + r_s)\frac{\pi_s \delta}{2(1 + \delta)}\frac{1}{1 - a}w.$$

6. Public information[11]

Public information or, equivalently, better private information, affects the information available to individuals and, as a consequence, the revision of their beliefs following the announced beliefs of their interlocutors. We show here, in a minimal example, that, as a consequence, public information may harm the ability of individuals to learn from the announced beliefs of others and that, at common knowledge, each individual may know less than he or she would have known had there been no public information.

Countries A and B are, along with other countries, members of a monetary union. A financial crisis, that has affected all countries in the union but A and B most severely, threatens its cohesion. At the juncture at which we are looking at the situation, there are five possible outcomes: either the union is preserved, but either country A or country B possibly exits the union, or the union is dissolved.

States of the world, ω, possible outcomes of negotiations, are, thus, as follows:

[11] See Dutta and Polemarchakis (2012).

- State (A, B): the union is preserved, and both countries A and B remain in the union;
- State (A, ~ B): the union is preserved, and country A remains in the union, but country B exits;
- State (~ A, B): the union is preserved, and country B remains in the union, but country A exits;
- State (~ A, ~ B): the union is preserved, but both countries A and B exit;
- State X: the union is dissolved.

All outcome are ex-ante equiprobable: $\pi(\omega) = 1/5$. Investors or policy-makers $i = 1, 2$ consider a project with state-contingent payoffs or value

$$v(\omega) = \begin{cases} 1 & if \quad \omega = (A, \sim B), (\sim A, B) \\ 0 & if \quad \omega = (A, B), (\sim A, \sim B), X \end{cases};$$

that is, the project pays off 1 in the asymmetric states (A, ~ B) or (~ A, B) and 0 otherwise. The structure of payoffs is designed as required; the ex ante expected payoff is $\mathrm{E}v(\omega) = 2/5$. The state of the world that obtains is (A, B): the union is preserved and both counties A and B remain in the union; ex post, the project is worth 0. However, the information of individuals concerning the outcome of the negotiations is incomplete: individual 1 learns whether the union is preserved and country A remains in the union or not; he or she learns nothing about the outcome concerning country B or about the state of the union if country A is not in it. That is, his or her information partition is

$$P^1 = \{\underbrace{\{(A, B), (A, \sim B)\}}_{P_1^1}, \underbrace{\{(\sim A, B), (\sim A, \sim B), X\}}_{P_2^1}\};$$

individual 2 learns whether the union is preserved and country B remains in the union or not; he or she learns nothing about the outcome concerning country A or about the state of the union if country B is not in it. That is, his or her information partition is

$$P^2 = \{\underbrace{\{(A, B), (\sim A, B)\}}_{P_1^2}, \underbrace{\{(A, \sim B), (\sim A, \sim B), X\}}_{P_2^2}\}.$$

Individual 1 is informed of P_1^1 and announces his or her posterior belief on the expected payoff of the project

$$v_1^1 = \mathrm{E}(v(\omega) \mid P_1^1) = \frac{1}{2}.$$

Individual 2 is informed of P_1^2, which does not allow him or her to know the information received by his or her interlocutor, P_1^1 or P_2^1, because both sets have non-null intersection with individual 2's information set. The information, P_1^1, received by individual 1, is, nevertheless, revealed by the announcement $v_1^1 = 1/2$, because $\mathrm{E}(v(\omega) \mid P_2^1) = 1/3$. This allows individual 2 to refine his or her information to

$$P_1^2(v_1^1) = P_1^2 \cap P_1^1 = \{(A, B)\}$$

and announce

$$v_1^2 = \mathrm{E}(v(\omega) \mid P_1^2(v_1^1)) = 0.$$

It is now the turn of individual 1 to reason that, had individual 2 been informed of P_2^2, and having heard $v_1^1 = 1/2$, he or she would have announced $\mathrm{E}(v(\omega) \mid P_2^2 \cap P_1^1) = 1$; individual 1 can also then refine his or her information to

$$P_2^1(v_1^1, v_1^2) = P_1^1 \cap P_1^2 = \{(A, B)\}$$

and announce

$$v_2^1 = \mathrm{E}(v(\omega) \mid P_2^1(v_1^1, v_1^2)) = 0.$$

At this point, the revision of beliefs ends: they are common knowledge, as in Geanakoplos and Polemarchakis (1982), and, as in Aumann (1976), they coincide. In addition, the information of individuals following the revision of their beliefs based on the announced beliefs of each other coincides with the information that would have been available to them had they exchanged information: $P_1^1 \cap P_1^2 = \{(A, B)\}$ is the element of the join of the individual partition that contains ω^*, the state that realized. It is argued in Geanakoplos and Polemarchakis (1982) that the announcement and revision of beliefs and the convergence to common knowledge and agreement does not imply convergence to the common beliefs the exchange of information implies.

Consider now a public announcement following the realization of ω'^* after the individuals have received their private information, but before the announcement and revision of beliefs: "$\omega \neq X$" or "the union has been preserved". This is something that both individuals know; it is public knowledge: $X \notin P_1^2 \cup P_1^1$. However, it is not common knowledge, and it affects the information that is communicated between the individuals by the announcement and revision of beliefs. Following (rather, in view of) the announcement, the information partitions of individuals are

$$P^1 = \{\underbrace{\{(A, B), (A, \sim B)\}}_{P_1^1}, \underbrace{\{(\sim A, B), (\sim A, \sim B)\}}_{P_2^1}, \underbrace{\{X\}}_{P_3^1}\},$$

and

$$P^2 = \{\underbrace{\{(A, B), (\sim A, B)\}}_{P_1^2}, \underbrace{\{(A, \sim B), (\sim A, \sim B)\}}_{P_2^2}, \underbrace{\{X\}}_{P_3^2}\}.$$

As earlier, individual 1 is informed of P_1^1 and announces his or her posterior belief

$$v_1^1 = \mathrm{E}(v(\omega) \mid P_1^1) = \frac{1}{2}.$$

However, this announcement no longer reveals the information of the individual: $\mathrm{E}(v(\omega) \mid P_1^1) = \mathrm{E}(v(\omega) \mid P_2^1) = 1/2$ and $P_1^2(v_1^1) = P_1^2$.

Individual 2 announces

$$v_1^2 = \mathrm{E}(v(\omega) \,|\, P_1^2(v_1^1)) = \frac{1}{2},$$

which, by a similar argument, reveals no information to individual 1. The revision of beliefs ends here: they are common knowledge, but they do not coincide with the information that would have been available to them had they exchanged information. This is reminiscent of the uncertainty that remains at a (common) beliefs equilibrium, the notion in Dutta and Morris (1997) that relaxes the more more restrictive notion of rational expectations. The generality and empirical relevance of the phenomenon remain to be looked into.

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