Macroeconomic equilibrium in a monetary economy [1] [2]

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Abstract

Consumption and investment are determined at equilibrium where a monetary authority sets rates of interest and accomodates the demand for money. Intertemporal substitution determines the impact of the nominal rate of interest on the rate of inflation, which differs between the short and the medium run. An increase in the rate of interest augments revenue from seignorage and, as a consequence, reduces the deficit of the public sector. Due to the cash - in - advance constraint, an increase in the rate of interest implies an increase in the effective price of consumption and, as a consequence, a decrease in consumption; the capital stock required decreases, and the effect applies retroactively to all dates, when the productive capacity is accumulated.

Key words: macroeconomic equilibrium; money.

JEL classification numbers: E00; F00.

1 Introduction

The keynesian apparatus [1] served long as the primary analytical tool for macoeconomic and monetary policy.

Following the new classical revival in macroeconomic theory [2], keynesian analysis fell in disrispute: it failed individual optimization and market clearing; under uncertainty, it failed rational expectations.

The difficulties encountered in providing equilibrium foundations for keynesian arguments stem from two, related, failures of the model of general competitive equilibrium [3]: the failure to account for the formation of prices and transactions out of equilibrium; and the failure to allow for money, a medium of exchange.

Fix - price equilibria [4] did not address the formation of prices, but did account for transactions away from competitive equilibrium prices with individual optimization and effective market clearing; they allowed for keynesian analysis [5], but the treatment of money and expectations remained unsatisfactory.

The conundrum associated with money in general equilibrium arise out of accounting considerations [6]: money, in positive supply, generates purchasing power that the initial holders of money wish to expend for the purchase of commodities; if the transactions of individuals satisfy an overall budget constraint, aggregation over individuals indicates that market clearing is not possible.

Equilibrium models of monetary economies consider situations in which an aggregate budget constraint need not be satisfied; this is the case in models of overlapping generations [7], where the infinity of individuals may prevent aggregation [8] or in models with cash - in - advance constraints [9] where, over an infinite horizon, a budget constraint may fail even for individuals [10]. Models of overlapping generations allow for interesting exercises in macroeconomic and monetary policy [11]; the multiplicity of equilibria that characterizes economies of overlapping generations may explain differences between keynesian and classical arguments and predictions concerning the effects of active policy [12]. Models with cash - in - advance constraints have provided the main analytical framework for the equilibrium analysis of macroeconomic and monetary policy [13]; but the literature has focused on explicit, recursive solutions that do not allow, among others, for heterogeneous agents, which limits the interest of the models.

[1] Hicks (1937), Keynes (1936).
[2] Lucas (1972).
[3] Arrow and Debreu (1954), McKenzie (1954).
[4] Benassy (1975), Drèze (1975).
[5] Malinvaud and Younès (1977).
[6] Hahn (1965).
[7] Allais (1947), Samuelson (1958).
[8] Geanakoplos and Polemarchakis(1990).
[9] Clower (1967).
[10] Grandmont and Younès (1972, 1973).
[11] Grandmont and Laroque (1975, 1976).
[12] Geanakoplos and Polemarchakis(1986).
[13] Lucas and Stokey (1987).

Banks that issue balances and distribute their profits as dividend to shareholders circumvent the distinction between inside and outside money and allow for money in general equilibrium [14]. Open market operations effect changes in the supply of money either as an autonomous policy choice or in order to accommodate demand at set rates of interest. Macroeconomic and monetary policy can be studied at equilibrium.

Modern monetary theory has focused away from the liquidity services provided by money and on the role of money as a unit of account. Monetary policy has real effects by modifying the structure of payoffs of assets when the asset market is incomplete [15] or the information revealed by prices when individuals are asymmetrically informed [16] The importance and interest of such considerations notwithstanding, the argument requires money to maintain a positive price at equilibrium.

Here, an elementary macroeconomic model of a monetary economy serves to illustrate the macroeconomic effects of monetary policy at equilibrium.

The central bank sets the rate of interest and supplies balances, through open market operations, in order to accomodate demand. Seignorage, transferred to the government, is the only source of revenue for public expenditures.

In particular, the impact of the rate of interest on the rate of inflation depends on the possibility of intertemporal substitution and differs between the short and the medium run. If the intertemporal distribution of output is fixed and, as a consequence, substitution is not possible, an increase in the rate of interest curbs inflation in the short run, but feeds inflation in the medium run; with intertemporal substitution and the real rate of interest determined by the real side of the economy, the rate of inflation increases with the rate of interest.

Since seignorage is transferred as revenue to the government, an increase in the rate of interest augments revenue from seignorage and, as a consequence, reduces the deficit of the public sector.

Due to the cash - in - advance constraint, an increase in the rate of interest implies an increase in the effective price of the consumption good and, as a consequence, a decrease in consumption; the capital stock required to produce the good decreases, and the effect applies retroactively to all dates, when the capacity required to produce the output at t is accumulated.

2 The economy

Dates are $t = 1, \ldots, T, T+1$.

Agents in the economy are the private sector, a representative consumer and a firm, and the public sector, the government and a central bank or monetary authority.

[14] Drèze and Polemarchakis (1998), Dubey and Geanakoplos (1992).

[15] Cass (1985), Geanakoplos and Mas - Colell (1989).

[16] Lucas (1972), Polemarchakis and Siconolfi (1983), Polemarchakis and Seccia (1998), Weiss (1980).

Exchange, production and consumption occur at dates $t = 1, \ldots, T$; date $t = T + 1$ serves only for purposes of accounting.

Commodities are a non - produced factor of production, labor, l_t, and output, y_t, that is consumed, c_t, or stored, k_t, if storage is possible.

The wage rate is w_t and the price of output is p_t.

The rate of inflation is $\pi_t = (p_{t+1}/p_t) - 1$.

The firm produces the consumption good at each date with labor as the only input according to the decreasing returns to scale technology

$$y_t = l_t^\alpha, \quad 0 < \alpha < 1, \quad l_t \geq 0.$$

The firm demands labor,

$$l_t = \alpha^{\frac{1}{1-\alpha}} (\frac{w_t}{p_t})^{\frac{1}{\alpha-1}},$$

supplies output,

$$y_t = \alpha^{\frac{\alpha}{1-\alpha}} (\frac{w_t}{p_t})^{\frac{\alpha}{\alpha-1}},$$

and generates profit,

$$v_t = (1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} (\frac{w_t}{p_t})^{\frac{\alpha}{\alpha-1}} p_t,$$

that it transfers to the consumer as dividend at date t, as it accrues.

The consumer is endowed with $\bar{l}_t$ units of labor that he supplies inelastically in order to consume. His utility function is

$$u(\ldots, c_t, \ldots) = \sum_{t=1}^{T} \beta^{t-1} \ln c_t, \quad \beta > 0, c_t > 0.$$

Nominal bonds, b_t serve to transfer purchasing power across dates. Bonds traded at date t are redeemed at date $t + 1$; the nominal rate of interest is r_t.

The real rate of interest is $\rho_t = ((1 + r_t)/(1 + \pi_t)) - 1$.

Money serves as a medium of exchange. The central bank sets the rate of interest and supplies balances, M_t, through open market operations in order to accommodate demand. The profit of the bank, seignorage $r_t M_t$, is transferred to the government at date $t + 1$, as it accrues.

The rate of growth of the money supply is $\mu_t = (M_{t+1}/M_t) - 1$.

The budget constraints of the consumer are

$$p_1 c_1 + b_1 + m_1 \leq w_1 \bar{l}_1 + v_1,$$

$$p_t c_t + b_t + m_t \leq w_t \bar{l}_t + (1 + r_{t-1})\, b_{t-1} + m_{t-1} + v_t, \quad t = 2, \ldots, T,$$

$$0 \leq (1 + r_T) b_T + m_T.$$

The liquidity constraints are

$$p_t c_t \leq \gamma m_t, \quad \gamma > 1,$$

where γ is the velocity of circulation of money.

The cumulative interest factor is $R_t = (1 + r_t) \times R_{t-1}$, with $R_0 = 1$.

The budget constraints of the consumer aggregate to

$$\sum_{t=1}^{T} \frac{p_t}{R_{t-1}} c_t + \sum_{t=1}^{T} \frac{r_t}{R_t} m_t \leq W,$$

where $W = \sum_{t=1}^{T} \frac{w_t}{R_{t-1}} \bar{l}_t + \sum_{t=1}^{T} \frac{v_t}{R_{t-1}} = \sum_{t=1}^{T} \frac{p_t y_t}{R_{t-1}}$ is the wealth of the individual discounted to date 1.

The demand for consumption by the private sector is

$$c_t = \frac{1-\beta}{1-\beta^T} \beta^{t-1} \frac{1}{p_t \alpha_t} W,$$

where

$$\alpha_t = \frac{\gamma (1 + r_t) + r_t}{\gamma R_t}.$$

The demand for money balances is

$$m_t = \frac{p_t c_t}{\gamma}.$$

The government does not levy taxes; the only source of revenue for the government is the segniorage transferred by the central bank. The government uses its revenue to finance public consumption, $g_t \geq 0$, and it incurs debt, d_t, in order to reallocate revenue across dates.

The budget constraints of the public sector are

$$p_1 g_1 + d_1 \leq 0,$$

$$p_t g_t + d_t \leq (1 + r_{t-1}) d_{t-1} + r_{t-1} M_{t-1}, \quad t = 2, \ldots, T,$$

$$0 \leq (1 + r_T) d_T + r_T M_T;$$

they aggregate to

$$\sum_{t=1}^{T} \frac{p_t}{R_{t-1}} g_t \leq \sum_{t=1}^{T} \frac{r_t}{R_t} M_t.$$

Government consumption at the terminal date can be interpreted as surplus of the public sector.

2.1 Exchange

Neither the consumption good nor labor are storable; the economy is, essentially, an economy of pure exchange and intertemporal substitution is not possible, which may reflect rigidities in investment.

Equilibrium requires that the goods market clears:

$$c_t + g_t = y_t,$$

the **IS** curve, as does the labor market:

$$l_t = \bar{l}_t;$$

the money market clears a fortiori, since the bank accommodates demand:

$$m_t = M_t,$$

the **LM** curve, while the bond market clears as a residual.

Lemma 1. *There exists an open set of competitive equilibrium paths of dimension $T + (T - 1)$, indexed by the path of nominal rates of interest, $r_1, \ldots, r_T$, the path of public consumption up to the terminal date, $g_1, \ldots, g_{T-1}$, and the price level at the initial date, p_1.*

Proof The argument is constructive.

The supply of output is determined by the supply of labor which is inelastic.

Public consumption at the terminal date, the surplus of the public sector, is determined by the solvency condition

$$0 \leq g_T = \frac{\frac{r_T}{\gamma(1+r_T)+r_T} + C}{1+C} y_T,$$

where

$$C = \sum_{t=1}^{T-1} \beta^{t-T} \frac{1}{y_t - g_t} \left(\frac{r_t}{\gamma(1+r_t)+r_t} y_t - g_t \right),$$

obtained from the demand by the private sector for consumption and money balances and the market clearing condition in the market for output.

With output and public consumption determined, private consumption is determined as a residual.

With the private consumption determined, the first order conditions for intertemporal optimization or, equivalently, the consumption demand function determine the path of prices of output up to the initial price level, p_1.

The initial price level can be set arbitrarily, with no effect on the equilibrium allocation.

The path of wages is determined by prices so that the demand for labor coincides with the inelastic supply.

With the path of prices and the path of private consumption determined, the demand for money by the individual is determined, which the monetary authority accommodates.

The construction is well defined as long as

$$0 < g_T < y_T.$$

Since rates of interest are non - negative, $g_T < y_T$; the condition $0 < g_T$ is satisfied for an open set of paths of rates of interest and government consumption up to the terminal date. $\square$

With the price level at the initial date, p_1, set, the central bank accomodates the demand for balances, M_1; alternatively, with the supply of balances at the initial date, M_1, set, the price level, p_1, adjusts.

Equilibria are distinct if the paths of individual consumption, $c_1, \ldots, c_T$ or equivalently, since output is exogenous, the paths of public consumption, $g_1, \ldots, g_T$ are distinct.

Proposition 1. *There exists an open set of distinct equilibrium allocations of dimension T, indexed by the path of public consumption up to the terminal date, $g_1, \ldots, g_{T-1}$ and the rate of interest at some date.*

Proof Since output is determined by the inelastic supply of labor and private consumption is determined as a residual, it suffices to consider the path of public consumption up to the terminal date, $g_1, \ldots, g_{T-1}$, as well as public surplus at the terminal date, g_T.

By proposition 1, government consumption at each date up to the terminal one enters as a free variable in the construction of equilibria.

It remains to show that the rate of interest at any date affects non - trivially public surplus at the terminal date; indeed, since, seignorage finances government consumption, an increase in the interest rate at any date results in an increase in public consumption at the terminal date,

$$\frac{\partial g_T}{\partial r_t} = \frac{y_T}{(1+C)^2} \beta^{t-T} \frac{\gamma^2 (1+r_T) y_t}{(\gamma(1+r_T)+r_T)(\gamma(1+r_t)+r_t)^2 (y_t - g_t)} > 0,$$

$$t = 1, \ldots, T-1,$$

$$\frac{\partial g_T}{\partial r_T} = \frac{\frac{\gamma}{(\gamma(1+r_T)+r_T)^2} y_T}{1+C} > 0,$$

this is the case. □

The allocation at equilibrium does not depend on the level of prices or, equivalently, the supply of money at the initial date; this is a consequence of the assumption of flexible prices.

Since output is exogenous, an alternative to the specification of public consumtion, $g_1, \ldots, g_{T-1}$, is the specification of consumtion, $c_1, \ldots, c_{T-1}$; in either case, the distribution of output at the terminal date must be determined endogenously, since there are no direct transfers of revenue between the private and the public sector - – seignorage is endogenously determined at equilibrium.

Rates of interest at all but one date fail to affect the allocation at equilibrium; this is due to the determination of individual consumption as a residual and the fixed level of output at each date resulting from the inelastic supply of labor and the absence of intertemporal transfers of output.

The rate of inflation along an equilibrium path is

$$\pi_t = \beta(1 + r_{t+1}) \frac{(\gamma(1 + r_t) + r_t)}{(\gamma(1 + r_{t+1}) + r_{t+1})} \frac{(y_t - g_t)}{(y_{t+1} - g_{t+1})} - 1, \quad t = 1, ..., T-1$$

while the rate of growth of the money supply is

$$\mu_t = \beta(1+r_{t+1})\frac{(\gamma(1+r_t)+r_t)}{(\gamma(1+r_{t+1})+r_{t+1})} - 1, \quad t = 1, ..., T-1$$

Proposition 2. *Across equilibrium paths, an increase in the rate of interest causes a decrease in the rate of inflation in the short run,*

$$\frac{\partial \pi_{t-1}}{\partial r_t} < 0, \quad t = 2, \ldots, T-1,$$

but an increase in the medium run,

$$\frac{\partial \pi_t}{\partial r_t} > 0, \quad t = 1, \ldots, T-2,$$

except for the terminal rate of interest, which does not affect the terminal rate of inflation,

$$\frac{\partial \pi_{T-1}}{\partial r_T} = 0.$$

Proof By direct computation, for $t = 2, \ldots, T-1$,

$$\frac{\partial \pi_{t-1}}{\partial r_t} = -\beta \frac{(\gamma(1+r_{t-1})+r_{t-1})}{(\gamma(1+r_t)+r_t)^2} \frac{(y_{t-1}-g_{t-1})}{(y_t-g_t)} < 0,$$

$$\frac{\partial \pi_t}{\partial r_t} = \frac{\beta(1+\gamma)(1+r_{t+1})}{(\gamma(1+r_{t+1})+r_{t+1})} \frac{(y_t-g_t)}{(y_{t+1}-g_{t+1})} > 0, \quad \text{and}$$

$$\frac{\partial \pi_t}{\partial r_s} = 0, \quad s = 1, \ldots, t-1, t+1, \ldots, T;$$

for $t = T-1$,

$$\frac{\partial \pi_{T-1}}{\partial r_s} = \beta(1+r_T)\frac{(\gamma(1+r_{T-1})+r_{T-1})}{(\gamma(1+r_T)+r_T)} \frac{(y_{T-1}-g_{T-1})}{(y_T-g_T)^2} \frac{\partial g_T}{\partial r_s} > 0,$$

$$s = 1, \ldots, T-2,$$

$$\frac{\partial \pi_{T-1}}{\partial r_{T-1}} =$$

$$\beta \frac{(1+\gamma)(1+r_T)}{(\gamma(1+r_T)+r_T)} \frac{(y_{T-1}-g_{T-1})}{(y_T-g_T)} +$$

$$\beta(1+r_T)\frac{(\gamma(1+r_{T-1})+r_{T-1})}{(\gamma(1+r_T)+r_T)} \frac{(y_{T-1}-g_{T-1})}{(y_T-g_T)^2} \frac{\partial g_T}{\partial r_{T-1}} > 0, \quad \text{and}$$

$$\frac{\partial \pi_{T-1}}{\partial r_T} = 0,$$

since

$$(\beta\frac{(\gamma(1+r_{T-1})+r_{T-1})}{(\gamma(1+r_T)+r_T)}\frac{(y_{T-1}-g_{T-1})}{(y_T-g_T)})^{-1}\frac{\partial\pi_{T-1}}{\partial r_T} =$$

$$\frac{-1}{(\gamma(1+r_T)+r_T)} + \frac{(1+r_T)}{(y_T-g_T)}\frac{\partial g_T}{\partial r_T} =$$

$$\frac{1}{(1+C)(\gamma(1+r_T)+r_T)(y_T-g_T)}(\frac{\gamma(1+r_T)y_T}{(\gamma(1+r_T)+r_T)} - (1+C)(y_T-g_T)) =$$

$$\frac{1}{(1+C)(\gamma(1+r_T)+r_T)(y_T-g_T)}$$

$$(\frac{\gamma(1+r_T)y_T}{\gamma(1+r_T)+r_T} - (1+C)y_T + y_T(\frac{r_T}{\gamma(1+r_T)+r_T} + C)) =$$

$$0.$$

□

Interest acts as a tax levied on consumers subject to a a cash - in - advance constraint: the effective price of consumption is $p_t((\gamma(1+r_t)+r_t)/\gamma R_t)$. In the absence of investment possibilities and, hence, of intertemporal substitution, individual consumption is determined as a residual, after output has been reduced by government consumption; the effective price of output adjusts for the the demand of the individual to coincide with the quantity available for consumption. An increase in the rate of interest, r_t, requires a reduction in the price of output, p_t, for the effective price of consumption to remain unaffected, and as a consequence, a decrease in the rate of inflation in the short run, π_{t-1}, but an increase in the rate of inflation in the medium run, π_t. The failure of the rate of interest to affect prices and rates of inflation in the long run, $s > t+1$, or to have retroactive effects, $s < t$, is an artifact of the separability implied by logarithmic preferences. The constancy of the terminal rate of inflation is harder to account for.

2.2 Investment

Alternatively, output is storable.

The investment technology, storage, is linear with productivity ρ. Output from storage is

$$\tilde{y}_{t+1} = (1+\rho)k_t, \quad t = 1, \ldots, T-1,$$

and investment is

$$i_t = k_{t+1} - k_t, \quad t = 2, \ldots, T-1,$$

with $i_1 = k_1$ and $k_T = 0$.

Profit maximization in investment requires that

$$1+\rho \geq \frac{1+r_t}{1+\pi_t}, \quad t = 1, \ldots, T-1,$$

with equality as long as the non - negativity constraint does not bind — in particular when $k_t > 0$. Equilibrium in the output market requires that

$$c_1 + g_1 + i_1 = y_1, \quad k_1 \geq 0,$$

$$c_t + g_t + i_t = y_t + \rho k_{t-1}, \quad k_t \geq 0, \quad t = 2, \ldots, T-1,$$

$$c_T + g_T = y_T + (1+\rho)k_{T-1},$$

the **IS** curve, while equilibrium conditions in other markets remain unchanged.

With positive investment at each date, the dimension of the set of equilibria remains unaffected, but the dimension of the set of distinct equilibrium allocations as well as comparative statics are different.

One restricts attention to values of the structural parameters of the economy — discount factor, β, the productivity of investment, ρ, the velocity of circulation of money, γ, the endowment of labor, $\bar{l}_t$, and the productivity of labor, α, such that equilibrium paths of the rate of interest, r_t, and public consumption, g_t, exist, with $k_t > 0$, for $t = 1, \ldots, T-1$.

Rates of interest and the productivity of investment determine the path of prices up to the initial price level,

$$p_t = \frac{R_{t-1}}{(1+\rho)^{t-1}} p_1, \quad t = 2, \ldots, T.$$

Private consumption at equilibrium is

$$c_t = \tfrac{(1-\beta)(\beta(1+\rho))^{t-1}}{(1-\beta^T)} \tfrac{\gamma(1+r_t)}{(\gamma(1+r_t)+r_t)} (\textstyle\sum_{s=1}^{T} \tfrac{y_s}{(1+\rho)^{s-1}}),$$

$$t = 1, \ldots, T.$$

Public consumption, surplus, at the terminal date is determined by the solvency condition

$$0 \leq g_T =$$

$$(1+\rho)^{T-1}((\textstyle\sum_{t=1}^{T} \tfrac{\beta^{t-1}(1-\beta)}{(1-\beta^T)} \tfrac{\gamma r_t}{\gamma(1+r_t)+r_t})(\textstyle\sum_{s=1}^{T} \tfrac{y_s}{(1+\rho)^{s-1}}) - \textstyle\sum_{t=1}^{T-1} \tfrac{g_t}{(1+\rho)^{t-1}}).$$

The capital stock at equilibrium is

$$k_t =$$

$$\textstyle\sum_{s=1}^{t}(1+\rho)^{t-s}(y_s - g_s - c_s), \quad t = 1, \ldots, T-1.$$

The price level depends on the initial stock of money,

$$p_1 = \frac{\gamma M_1}{y_1 - g_1 - k_1},$$

while the rate of growth of the supply of money supply is

$$\frac{M_{t+1}}{M_t} = \frac{p_{t+1}c_{t+1}}{p_t c_t} = \beta(1+r_{t+1}) \frac{\gamma(1+r_t) + r_t}{\gamma(1+r_{t+1}) + r_{t+1}}.$$

Proposition 3. *There exists an open set of distinct equilibrium allocations of dimension $T + (T-1)$, indexed by the path of government consumption up to the terminal date, $g_1, \dots, g_{T-1}$, and the path of rates of interest, $r_1, \dots, r_T$.*

Proof Government consumption up to the terminal date enter as independent variables in the construction of equilibria.

It remains to show that the rate of interest at every date affects non - trivially individual consumption at some date; indeed, since

$$\tfrac{\partial c_t}{\partial r_t} = \tfrac{-\gamma(1-\beta)(\beta(1+\rho))^{t-1}}{(1-\beta^T)} \tfrac{1}{(\gamma(1+r_t)+r_t)^2} (\textstyle\sum_{s=1}^T \tfrac{y_s}{(1+\rho)^{s-1}}) < 0,$$

$$t = 1, \dots, T.$$

this is the case. □

With the real rate of interest determined by the investment technology, the nominal rate of interest has a simple effect on the rate of inflation,

$$\frac{\partial \pi_t}{\partial r_s} = \begin{cases} (1+\rho)^{-1} > 0, & \text{if } s = t, \\ \\ 0, & \text{otherwise.} \end{cases}$$

An increase in the rate of interest has an immediate, depressing impact on consumption, $\partial c_t / \partial r_t < 0$, and no retroactive or medium or long term effects.

Government consumption at the terminal date increases with the rate of interest at any date,

$$\tfrac{\partial g_T}{\partial r_t} = (1+\rho)^{T-1} \tfrac{\beta^{t-1}\gamma(1-\beta)}{(1-\beta^T)} \tfrac{\gamma}{(\gamma(1+r_t)+r_t)^2} (\textstyle\sum_{s=1}^T \tfrac{y_s}{(1+\rho)^{s-1}}) > 0,$$

due to the positive effect of the rate of interest on seignorage and, hence, on government revenue.

Proposition 4. *Across equilibrium paths, an increase in the rate of interest at date t leads to an increase in the capital at dates $s = 1, \dots, t$.*

Proof By direct computation,

$$\tfrac{\partial k_t}{\partial r_s} =$$

$$-\tfrac{\gamma(1-\beta)\beta^{s-1}(1+\rho)^{t-1}}{(1-\beta^T)} \tfrac{\gamma}{(\gamma(1+r_s)+r_s)^2} (\textstyle\sum_{j=1}^T \tfrac{y_j}{(1+\rho)^{j-1}}) < 0,$$

$$s = 1, \dots, t, \quad t = 1, \dots, T-1.$$

An increase in the rate of interest implies an increase in the effective price of the consumption good and, as a consequence, a decrease in consumption at date t; the capital stock required to produce the good decreases, and the effect applies retroactively to all prior dates, when the capacity required to produce the output at t is accumulated.

It is of interest to consider equilibrium paths along which the non - negativity of the capital stock, $k_t \geq 0$, or, alternatively, a non - negativity constraint on investment, $i_t \geq 0$, is a binding constraint; equivalently, equilibrium paths that are characterized by excess productive capacity. Along such paths, the comparative statics switch between the comparative statics of an exchange economy, at dates with a binding non - negativity of investment, and the comparative statics of an economy with production, at dates with positive investment.

3 Conclusions and extensions

The model extends to encompass economies under uncertainty, in which the role of money as a unit of account is fundamental.

In economies with nominal rigidities, in which the price mechanism does not attain market clearing or an optimal allocation of resources, the argument for active monetary policy is compelling and it is of interest to characterize improving interventions.

In an economy with multiple currencies, the exchange rate may substitute the rate of interest as the instrument of monetary policy [17].

[17] Mundell 1963

References

Allais, M. (1947), *Economie at interêt,* Imprimerie Nationale (France).

Arrow, K. J. and G. Debreu (1954), "The existence of equilibria for a competitive economy, "*Econometrica,* 22, 265 - 290.

Benassy, J. - P. (1975), "Neo - keynesian equilibrium theory in a monetary economy, "*Review of Economic Studies,* 42, 503 - 523.

Cass, D. (1985), "On the 'number' of equilibrium allocations with finacial markets, "Working Paper No. 8500, CARESS, University of Pennsylvania.

Clower, R. (1967), "A reconsideration of the microfoundations of monetary theory, "*Western Economic Journal,* 6, 1 - 8.

Drèze (1975), "Existence of an exchange equilibrium under price rigidities, "*International Economic Review,* 37, 1457 - 1482.

Drèze, J. H. and H. M. Polemarchakis (1998), "Monetary equilibria, "Discussion Paper No. 0000, CORE, Universit/'e Catholique de Louvain.

Dubey, P. and J. D. Geanakoplos (1992), "The value of money in a finite horizon economy: a role for banks, "in P. S. Dasgupta, D. Gale, O. D. Hart and E. Maskin (eds.) *Economic Analysis of Markets and Games, Essays in Honor of Frank Hahn,* M.I.T. Press, 407 - 444.

Geanakoplos, J. D. and A. Mas - Colell (1989), "Real indeterminacy with financial assets, "*Journal of Economic Theory*, 47, 22 - 38.

Geanakoplos, J. D. and H. M. Polemarchakis (1986), "Walrasian indeterminacy and keynesian macroeconomics, "*Review of Economic Studies,* 00, 000 - 000.

Geanakoplos, J. D. and H. M. Polemarchakis (1990), "Overlapping generations, "in W. Hildenbrand and H. Sonnenschein (eds.), *Handbook of Mathematical Economics*, North Holland, 000 - 000.

Grandmont, J. - M. and G. Laroque (1975), "On money and banking, "*Review of Economic Studies,* 42, 207 - 236.

Grandmont, J. - M. and G. Laroque (1976), "The liquidity trap, "*Econometrica,* 44, 129 - 135.

Grandmont, J. - M. and Y. Younès (1972), "On the role of money and the existence of a monetary equilibrium, "*Review of Economic Studies,* 39, 355 - 372.

Grandmont, J.M. and Y. Younès (1973), "On the efficiency of monetary equilibrium, "*Review of Economic Studies,* 40, 149 - 165.

Hahn, F. H. (1965), “On some problems in proving the existence of an equilibrium in a monetary economy, ”in F. H. Hahn and F. P. R. Brechling (eds), *The Theory of Interest Rates*, Macmillan, 000 - 000.

Hicks, J. R. (1937), “Mr. Keynes and the classics, ”*Econometrica*, 5, 147 - 159.

Keynes, J. M. (1936), *The General Theory of Employment, Interest and Money,* Macmillan.

Lucas, R.E. (1972), “Expectations and the neutrality of money, ”*Journal of Economic Theory*, 4, 103 - 124.

Lucas, R.E. and N.L. Stokey (1987), “Money and rates of interest in a cash - in - advance economy, ”*Econometrica,* 55, 491 - 513.

McKenzie, L. (1954), “Equilibrium in Graham’s model of world trade and other competitive systems, ”*Econometrica*, 22, 147 - 161.

Malinvaud, E. and Y. Younès (1977), “Une nouvelle formulation générale pour l’etude des fondements microéconomiques de la macro - économie, ”*Cahiers du Séminaire d’Econométrie,* 00, 000 - 000.

Mundel, R. (1963), “Capital mobility and stabilization policy under fixed and flexible exchange rates, ”*Canadian Journal of Economics,* 00, 000 - 000.

Polemarchakis, H. M. and G. Seccia (1998), “A role for monetary policy when prices reveal information: an example, ”mimeo.

Polemarchakis, H. M. and P. Siconolfi (1993), “Asset markets and the information revealed by prices, ”*Economic Theory,* 3, 645 - 661.

Samuelson, P. A. (1958), “An exact consumption loan model of interest with or without the social contrivance of money, ”*Journal of Political Economy,* 66, 467 - 482.

Weiss, L. (1980), “The role of monetary policy with rational expectations, ”*Journal of Political Economy,* 88, 221 - 233.