



# Short sales, destruction of resources, welfare<sup>☆</sup>



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## ABSTRACT

A reduction in the output of productive assets (trees) in some contingencies may expand the range of risks spanned by the payoffs of assets and allow for better risk sharing; which may compensate for the loss of output and support a Pareto superior allocation. Surprisingly, if short sales of assets are not allowed, improved risk sharing that results from the destruction of output does not suffice to support a Pareto superior allocation.

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## 1. Introduction

In an economy with uncertainty and limited markets for the reallocation of risks, the destruction of output may be Pareto improving. Typically, competitive allocations with an incomplete asset market are not Pareto optimal. A reduction in the output of productive assets (trees) in some states of the world can alter, in particular expand the span of the payoffs of assets; and, improved risk sharing may compensate for the loss of output and support a Pareto superior allocation.

We show here that, surprisingly, if short sales of assets are not allowed, improved risk sharing that results from the destruction of output does not suffice to induce a Pareto superior allocation.

In a renown contribution, [Aumann and Peleg \(1974\)](#) pointed out, through an elementary example, that an individual may benefit from the destruction of some of his endowment: the change in the terms of trade in response to the reduction in resources may be in his favor, and the benefit may compensate for the loss of revenue.<sup>1</sup> In a first-best environment, the individual benefits at the expense of others.<sup>2</sup> Our argument, here, is that under uncertainty,

an incomplete asset market and a ban on short sales, Pareto improvement is still not possible; an individual may benefit from the destruction of output in some states, but only at the expense of others; this, even though a Pareto improvement would be possible if assets could be traded with no restrictions.

Our paper is not the first to consider economies with productive assets or a ban on short sales. [Santos and Woodford \(1997\)](#) proved that, if traded assets are sufficiently productive in the sense that the aggregate endowment is bounded by a portfolio trading plan, then, pricing bubbles do not occur for securities in positive net supply, regardless of the presence of sequentially incomplete markets, arbitrary borrowing limits and incomplete participation of households in the infinite sequence of spot markets. [Demange \(2002\)](#) proved that, in an economy of overlapping generations with life-spans of two dates and one commodity at each date, land and no short sales of land guarantee the constrained optimality of competitive allocations. [Lucas \(1978\)](#) examined the behavior of equilibrium of asset prices in an economy with productive assets and no short sales.

First, we give an example to show that the destruction of output can indeed be Pareto improving when short-sales are allowed and, also, that destruction of output can augment insurance opportunities when short sales are prohibited. Subsequently, we show, in a general context, that destruction is never Pareto improving under no short sales. Finally, we argue that, still without short sales, an individual may benefit from the destruction of some of the output of assets he is endowed with, even though this reduction in resources is not Pareto-improving.

## 2. Example

In a particular environment, first, with unrestricted asset trades (short sales are allowed), destruction of output may support a

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<sup>1</sup> Effectively, the agent behaves strategically, and, as a monopolist would do, he may restrict supply to benefit from an increase in price.

<sup>2</sup> The argument is not unrelated to the transfer paradox in [Leontief \(1936\)](#) and the long and contentious literature that followed. [Donsimoni and Polemarchakis \(1994\)](#) generalized the result by showing that the distribution of welfare gains and losses resulting from the transfer of resources across individuals is unrestricted; the same holds for the destruction of resources.

Pareto improvement; second, with a ban on short sales, destruction never supports a Pareto improvement, and this even though it augments insurance opportunities.

Dates are 0 and 1, and two equiprobable states of the world,  $s = a, b$ , realize at 1; one perishable commodity,  $c$ , is exchanged and consumed only at date 1; two individuals,  $i = 1, 2$ , have identical utility functions

$$U^i = E \ln(c_s^i);$$

and two assets (trees),  $j = 1, 2$ , with identical risk-free dividends, equal to 1, each in unit net supply are each held by a different individual;  $z_j^i$  are holdings of assets. Endowments in commodities are

$$(e_a^1, e_b^1) = (1, 0), \quad (e_a^2, e_b^2) = (0, 1).$$

Since the assets have identical dividend patterns, no risk sharing is possible and, at equilibrium, there is no trade: the equilibrium is autarkic, and the allocation coincides with the allocation of endowments augmented with dividends:

$$(c_a^1, c_b^1) = (2, 1), \quad (c_a^2, c_b^2) = (1, 2).$$

### 2.1. Pareto improving destruction

If  $\epsilon > 0$  units of the output of tree 1 are destroyed at state  $b$ , the optimization problems, now different across individuals, are

$$\begin{aligned} \max_{c,z} E \log(c_s^1) & \quad \max_{c,z} E \log(c_s^2) \\ \text{s.t. } q_1 z_1^1 + q_2 z_2^1 = q_1 & \quad \text{and} \quad \text{s.t. } q_1 z_1^2 + q_2 z_2^2 = q_2 \\ c_a^1 = 1 + z_1^1 + z_2^1 & \quad c_a^2 = z_1^2 + z_2^2 \\ c_b^1 = (1 - \epsilon)z_1^1 + z_2^1 & \quad c_b^2 = 1 + (1 - \epsilon)z_1^2 + z_2^2, \end{aligned}$$

with prices of assets  $q_j$  and the commodity as numéraire.

Since the asset market is complete, the economy reduces to an economy with trades in contingent commodities,

$$\begin{aligned} \max_c E \log(c_s^1) & \quad \max_c E \log(c_s^2) \\ \text{s.t. } & \quad \text{and} \quad \text{s.t.} \\ p_a c_a^1 + p_b c_b^1 = p_a 2 + p_b (1 - \epsilon), & \quad p_a c_a^2 + p_b c_b^2 = p_a + p_b 2, \end{aligned}$$

with  $p_s$  contingent commodity prices.

The equilibrium allocation is

$$\begin{aligned} c_a^{*1} &= \frac{9 - 5\epsilon}{6 - 2\epsilon}, & c_b^{*1} &= \frac{9 - 5\epsilon}{6}, \\ c_a^{*2} &= \frac{9 - \epsilon}{6 - 2\epsilon}, & c_b^{*2} &= \frac{9 - \epsilon}{6}. \end{aligned}$$

As  $\epsilon \rightarrow 0$ , the allocation approaches 3/2, the full insurance allocation. It follows that, for small  $\epsilon$ , destruction induces a strict Pareto improvement.

The allocation can be supported as a sequential equilibrium allocation. Asset demands, by substituting the  $c^*$  allocation into the budget constraints of the sequential problem, are

$$\begin{aligned} z_1^1 &= -\frac{1}{\epsilon} \left( 1 + \frac{9 - 5\epsilon}{6} - \frac{9 - 5\epsilon}{6 - 2\epsilon} \right), \\ z_2^1 &= \frac{1}{\epsilon} \left( \frac{9 - 5\epsilon}{6} - \frac{3 - 3\epsilon}{6 - 2\epsilon} \right), \end{aligned}$$

and, by market clearing,  $z_1^2 = 1 - z_1^1$  and  $z_2^2 = 1 - z_2^1$ . For  $0 < \epsilon < 1$ ,<sup>3</sup>  $z_1^1 < 0$  and  $z_2^1 > 1$ . To support the allocation as a sequential equilibrium, individuals have to sell short. Finally, asset prices are computed from the first order conditions of the sequential problem by substituting the  $c^*$  allocation.

**Remark 1.** Here, it is the competitive allocation, following the destruction of output, that implements a Pareto improvement. This follows from the autarky that characterizes the competitive equilibrium before destruction. With destruction and improved risk sharing, Pareto superior allocations exist generically; but, it need not be a (the) competitive allocation that is Pareto superior.

### 2.2. Ban on short sales

If  $\epsilon > 0$  units of the output of tree 1 are destroyed at state  $b$ , but, importantly, short sales are not allowed, the optimization problems are

$$\begin{aligned} \max_{c,z} E \log(c_s^1) & \quad \max_{c,z} E \log(c_s^2) \\ \text{s.t. } q_1 z_1^1 + q_2 z_2^1 = q_1 & \quad \text{s.t. } q_1 z_1^2 + q_2 z_2^2 = q_2 \\ c_a^1 = 1 + z_1^1 + z_2^1 & \quad \text{and} \quad c_a^2 = z_1^2 + z_2^2, \\ c_b^1 = (1 - \epsilon)z_1^1 + z_2^1 & \quad c_b^2 = 1 + (1 - \epsilon)z_1^2 + z_2^2 \\ z_1^1, z_2^1 \geq 0, & \quad z_1^2, z_2^2 \geq 0; \end{aligned}$$

the last constraint in each bans short sales.

There is an equilibrium with the following properties: the short sales constraint on  $z_1^1$  binds, and  $z_1^1 = 0$ , while  $z_1^2 = 1$ ;  $z_2^i \in (0, 1)$ ; individual 2 insures against the realization of state  $a$  (the bad state, for 2) by transferring wealth from state  $b$  to state  $a$ , but individual 1 becomes worse-off relative to the (autarkic) equilibrium without destruction of resources; destruction introduces insurance opportunities not previously available.

The optimality conditions for  $z_1^1$  and  $z_2^1$ , respectively, are

$$\frac{1}{2} \frac{1}{c_a^1} + \frac{1}{2} \frac{1 - \epsilon}{c_b^1} \leq \lambda_0^1 q_1, \quad \text{and} \quad \frac{1}{2} \frac{1}{c_a^1} + \frac{1}{2} \frac{1}{c_b^1} = \lambda_0^1 q_2; \quad (1)$$

$\lambda_0^1$  is the marginal utility of revenue at date 0; similarly, for  $z_1^2$  and  $z_2^2$ ,

$$\frac{1}{2} \frac{1}{c_a^2} + \frac{1}{2} \frac{1 - \epsilon}{c_b^2} = \lambda_0^2 q_1, \quad \text{and} \quad \frac{1}{2} \frac{1}{c_a^2} + \frac{1}{2} \frac{1}{c_b^2} = \lambda_0^2 q_2. \quad (2)$$

Substitution of  $z_1^1 = 0$  and  $z_1^2 = 1$  into the date 0 budget constraints of each individual gives

$$z_2^1 = \frac{q_1}{q_2}, \quad \text{and} \quad z_2^2 = 1 - \frac{q_1}{q_2}. \quad (3)$$

Substitution of the optimality conditions (2) into the date 0 constraint of individual 2 and rearranging terms gives

$$\frac{z_2^2}{1 + z_2^2} = \frac{\epsilon - z_2^2}{2 - \epsilon + z_2^2}$$

or, equivalently,

$$z_2^2 = \frac{-3 + 2\epsilon \pm \sqrt{9 - 4\epsilon + 4\epsilon^2}}{4}.$$

For  $0 < \epsilon < 1$  only one root satisfies  $z_2^2 \in (0, 1)$ ; and, it is easy to verify that  $z_2^2 < \epsilon$  and, by market clearing,  $z_2^1 > 1 - \epsilon$ .

Evidently, asset holdings pin down consumption allocations.

Finally, after substitution for  $q_1$  into (1), the optimality condition (inequality) for  $z_1^1$  gives

$$\frac{1}{2 - z_2^2} + \frac{1 - \epsilon}{1 - z_2^2} \leq \frac{\frac{1}{2 - z_2^2} + \frac{1}{1 - z_2^2}}{\frac{1}{1 + z_2^2} + \frac{1}{2 - \epsilon + z_2^2}} \left( \frac{1}{1 + z_2^2} + \frac{1 - \epsilon}{2 - \epsilon + z_2^2} \right). \quad (4)$$

Numerically, it can be verified that, for  $0 < \epsilon < 1$ , (4) is satisfied with strict inequality and, as a result, the no short sales constraint for  $z_1^1$  is binding. This result follows from the fact that individual 1,

<sup>3</sup> We assume throughout the paper that dividends after destruction are positive.

when short sales are allowed, always short sells asset 1 and buys asset 2 for any  $0 < \epsilon < 1$ .

Individual 2 insures against the realization of state  $a$  relative to the autarkic allocation, where there is no risk sharing, and benefits in terms of welfare; in particular,  $c_a^2 = 1 + z_2^2 > 1$  and  $c_b^2 = 2 - (\epsilon - z_2^2) < 2$ . On the other hand, individual 1 is worse off relative to the autarkic allocation:  $c_a^1 = 1 + z_2^1 < 2$  and  $c_b^1 = z_2^1 < 1$ .

**Remark 2.** Destruction can augment insurance opportunities even in the absence of short sales. Nevertheless, it cannot implement a Pareto improvement; and this, at any feasible allocation, as the general argument that follows demonstrates.

**Remark 3.** Incentives for individuals to destroy part of the payoffs of assets they hold are relevant, even if not the focus in this paper. Above, individual 1, who is endowed with asset 1, does not have incentives to destroy. In Section 4, we construct an example, with three states of the world, in which individuals do have incentives to destroy—at the expense of others.

**3. The general argument**

Dates are 0 and 1, and states of the world,  $s = 1, 2, \dots, S$ , realize at date 1 with probability  $\pi_s$ ; one perishable commodity,  $c$ , is exchanged and consumed only at date 1; individuals,  $i = 1, 2, \dots, I$ , have utility functions

$$U^i = Eu^i(c_s^i),$$

with cardinal indices that are differentially strictly monotonically increasing and strictly concave and satisfy boundary conditions that allow us to restrict attention to strictly positive consumption allocations; and endowments in commodities  $e_s^i > 0$ .

Assets (trees),  $j = 1, 2, \dots, J$ , in unit aggregate supply, yield dividend payoffs  $d_s^j > 0$ ; the  $S \times J$  matrix of payoffs is  $D$ . Investment in assets are  $z_j^i$ , while  $1 \geq \bar{z}_j^i \geq 0$  are endowments in assets.

An allocation of assets defines an allocation of consumption, and we use the term allocation to refer to either. It is constrained Pareto optimal if no feasible allocation yields a Pareto superior allocation—constrained optimal, since the asset market may be incomplete.

Destruction of dividends may increase the rank of  $D$  and, as in the example, expand insurance opportunities as long as

$$\text{rank } D < J \leq S.$$

The decision problem of an individual is

$$\begin{aligned} \max_{c, z} \quad & U^i = Eu^i(c_s^i) \\ \text{s.t.} \quad & \sum_j q_j z_j^i = \sum_j q_j \bar{z}_j^i \\ & c_s^i = e_s^i + \sum_j d_s^j z_j^i \\ & z_j^i \geq 0, \end{aligned}$$

where  $q_j$  are prices of assets; evidently, the last constraint bans short sales.

First order conditions are

$$\begin{aligned} E[d_s^j u^i(c_s^i)] - \lambda_0^i q_j &\leq 0, \\ z_j^i (E[d_s^j u^i(c_s^i)] - \lambda_0^i q_j) &= 0; \end{aligned}$$

$\lambda_0^i$  is the marginal utility of revenue at date 0.

A competitive equilibrium consists of an allocation and prices, such that individuals maximize and markets clear:

$$\sum_i z_j^i = 1;$$

equivalently, the allocation is feasible.

Competitive equilibria exist<sup>4</sup>; competitive equilibrium allocations are constrained Pareto optimal, and any constrained Pareto optimal allocation is a competitive equilibrium allocation for some distribution of endowments of assets.<sup>5</sup> As a consequence, we think interchangeably of competitive and constrained optimal allocations.

An equilibrium with a ban on short sales is  $\{z_j^i, q_j\}$ . Subsequently, part of the dividends of each/some assets in each/some states are destroyed:  $e_s^i$ , with dividends reduced to  $\tilde{d}_s^i = d_s^i - \epsilon_s^i > 0$ . Short sales on all assets are still not allowed, and a feasible allocation after destruction is  $\{\tilde{c}^i, \tilde{z}_j^i\}$ .

**Proposition 1.** *Absent short sales, a feasible allocation following the reduction in dividends cannot be Pareto improving, the augmented insurance opportunities notwithstanding.*

**Proof.** We argue by contradiction. Suppose  $\{\tilde{z}^i\}$  dominates  $\{z^i\}$ , that is,

$$\Delta U^i = Eu^i(\tilde{c}_s^i) - Eu^i(c_s^i) \geq 0, \tag{5}$$

with strict improvement for some.

The differential property of concavity implies that

$$\Delta U^i < E[u^i(c_s^i)(\tilde{c}_s^i - c_s^i)]. \tag{6}$$

We define the right hand side of (6) as  $\Phi^i$ . It follows from (5) and (6) that  $\Phi^i$  should be positive for all individuals. Substituting for asset holdings into  $\Phi^i$ , gives

$$\Phi^i = E\left[u^i(c_s^i) \left(\sum_j \tilde{d}_s^j \tilde{z}_j^i - \sum_j d_s^j z_j^i\right)\right].$$

Substituting the first order conditions for optimization at the initial allocation (before destruction) gives

$$\frac{\Phi^i}{\lambda_0^i} = -\sum_j z_j^i q_j + E\left[\sum_j \frac{u^i(c_s^i)}{\lambda_0^i} \tilde{d}_s^j \tilde{z}_j^i\right];$$

taking the sum over  $i$  and, since assets are in unit supply,

$$\sum_i \frac{\Phi^i}{\lambda_0^i} = -\sum_j q_j + E\left[\sum_j \tilde{d}_s^j \sum_i \frac{u^i(c_s^i)}{\lambda_0^i} \tilde{z}_j^i\right]. \tag{7}$$

To complete the proof and derive a contradiction it suffices to show that the sum is negative that, in turn, implies that (6) is violated for some  $i$ . The first order conditions at the initial competitive equilibrium,  $E[d_s^j u^i(c_s^i)] - \lambda_0^i q_j \leq 0$ , yield, after multiplication by  $\tilde{z}_j^i \geq 0$ , summation over  $i$ , and since assets are in unit net supply,

$$E\left[d_s^j \sum_i \frac{u^i(c_s^i)}{\lambda_0^i} \tilde{z}_j^i\right] - q_j \leq 0. \tag{8}$$

Taking into account (8),  $d_s^j > \tilde{d}_s^j$ , and, importantly,  $\tilde{z}_j^i \geq 0$  (which guarantees that each term inside the expectation operator in (7) and (8) is non-negative), it follows that the sum in (7) is negative. This completes the argument.  $\square$

The intuition behind Proposition 1 is straightforward. The change in the welfare of an investor decomposes into two parts: one due to the change in the portfolio he holds, and the other due to the change in the payoffs of assets. By the envelop theorem, at equilibrium (or, equivalently, a constrained optimum), the first

<sup>4</sup> Magill and Quinzii (1996) is the definitive reference.

<sup>5</sup> Diamond (1967) and, more abstractly, Radner (1974) give the argument.

term adds up to 0 across individuals. With positive holdings of assets and only reduction in the payoffs, the second is non-positive for each individual, each asset and each state. It follows that a Pareto improvement is not possible.

**Remark 4.** The model incorporates the case of date 0 consumption; relabel one of the states as date 0 consumption and rearrange the budget constraints accordingly.<sup>6</sup> Also, state-dependent cardinal utility indices do not interfere with the argument.

**Remark 5.** A question that arises is whether Pareto improvement via the destruction of output is possible when short sales are restricted, but not prohibited: asset holdings are simply bounded below. It is, but the gains converge to 0 with the lower bound.

**4. Extensions: incentives and intervention**

Even though the destruction of output cannot be Pareto improving if short sales are not allowed, an individual may benefit from the destruction of some of the output in his endowment. As in [Aumann and Peleg \(1974\)](#), an individual can effectively exercise market power and benefit from the increase in the price of goods that he supplies following the destruction of part of the output. More interestingly, however, here, he can benefit from the insurance opportunities the assets in his endowment offer after the destruction of dividends. An example makes the point.

Dates are 0 and 1, and two equiprobable states of the world,  $s = a, b$ , realize at 1; one perishable commodity,  $c$ , is exchanged and consumed at each date-event; two individuals,  $i = 1, 2$ , have identical utility functions

$$U^i = c_0^i + E \frac{(c_s^i)^{1-\gamma}}{1-\gamma}, \quad \gamma > 0;$$

and two assets (trees),  $j = 1, 2$ , with identical risk-free dividends, equal to one, each in unit net supply are held by individual 2;  $z_j^i$  are holdings of assets. Endowments in commodities are

$$(e_a^1, e_b^1) = (1, 0), \quad (e_a^2, e_b^2) = (0, 1),$$

and  $e_0^i > 0$  date 0 endowments.

The quasi-linearity of the utility functions, equiprobable states, and the endowment pattern imply that no destruction date 1 equilibrium allocations are

$$(c_a^1, c_b^1) = (2, 1), \quad (c_a^2, c_b^2) = (1, 2),$$

and for appropriate specifications of  $e_0^i$ , consumption at date 0 is positive.

If  $\epsilon > 0$  units of the output of tree 1 are destroyed at state  $b$ , the optimization problems, now different across individuals, are

$$\begin{array}{ll} \max_{c,z} & c_0^1 + E \frac{(c_s^1)^{1-\gamma}}{1-\gamma} & \max_{c,z} & c_0^2 + E \frac{(c_s^2)^{1-\gamma}}{1-\gamma} \\ \text{s.t} & & \text{s.t} & \\ c_0^1 + q_1 z_1^1 + q_2 z_2^1 = e_0^1 & \text{and} & c_0^2 + q_1 z_1^2 + q_2 z_2^2 = e_0^2 + q_1 + q_2 & \\ c_a^1 = 1 + z_1^1 + z_2^1 & & c_a^2 = z_1^2 + z_2^2 & \\ c_b^1 = (1-\epsilon)z_1^1 + z_2^1 & & c_b^2 = 1 + (1-\epsilon)z_1^2 + z_2^2 & \\ z_1^1, z_2^1 \geq 0, & & z_1^2, z_2^2 \geq 0. & \end{array}$$

There is an equilibrium where the short sales constraint on  $z_1^1$  binds (and in equilibrium  $z_2^1 = 1$ ) and  $z_2^2 \in (0, 1)$ . Individual 2 becomes better off relative to the no destruction equilibrium

whereas individual 1 worse off. Effectively, individual 2 has an incentive to destroy part of the output in his endowments.

The optimality conditions for  $z_1^1$  and  $z_2^1$ , respectively, are

$$\begin{aligned} \frac{1}{2}(c_a^1)^{-\gamma} + \frac{1-\epsilon}{2}(c_b^1)^{-\gamma} - q_1 &\leq 0, \quad \text{and} \\ \frac{1}{2}(c_a^1)^{-\gamma} + \frac{1}{2}(c_b^1)^{-\gamma} &= q_2 \end{aligned} \tag{9}$$

and for  $z_1^2$  and  $z_2^2$ , respectively, are

$$\begin{aligned} \frac{1}{2}(c_a^2)^{-\gamma} + \frac{1-\epsilon}{2}(c_b^2)^{-\gamma} &= q_1, \quad \text{and} \\ \frac{1}{2}(c_a^2)^{-\gamma} + \frac{1}{2}(c_b^2)^{-\gamma} &= q_2. \end{aligned} \tag{10}$$

Combining the first order conditions for  $z_2^1$  and  $z_2^2$  from (9) and (10) and substituting budget constraints and market clearing into them, we obtain

$$(1 + z_2^1)^{-\gamma} + (z_2^1)^{-\gamma} = (2 - z_2^1)^{-\gamma} + (3 - \epsilon - z_2^1)^{-\gamma}; \tag{11}$$

a solution  $z_2^1 \in (0, 1)$  to (11) exists. Combining the first order conditions for  $z_1^1$  and  $z_1^2$ , we obtain

$$\begin{aligned} (1 + z_2^1)^{-\gamma} + (1 - \epsilon)(z_2^1)^{-\gamma} - (2 - z_2^1)^{-\gamma} \\ - (1 - \epsilon)(3 - \epsilon - z_2^1)^{-\gamma} &\leq 0; \end{aligned} \tag{12}$$

(12) must be satisfied with strict inequality for the short sales constraint on  $z_1^1$  to bind.

Individual 2 has an incentive to destroy if and only if

$$\begin{aligned} \Delta U^2 = \frac{(2 - z_2^1)^{1-\gamma}}{1-\gamma} + \frac{(3 - \epsilon - z_2^1)^{1-\gamma}}{1-\gamma} - \frac{1}{1-\gamma} - \frac{2^{1-\gamma}}{1-\gamma} \\ + z_2^1 \left( (2 - z_2^1)^{-\gamma} + (3 - \epsilon - z_2^1)^{-\gamma} \right) - 1 - 2^{-\gamma} > 0, \end{aligned} \tag{13}$$

where  $\Delta U^2$  denotes the difference in utility between destruction and no destruction.

To facilitate computations, fix  $\gamma = 4$  and  $\epsilon = 5 \times 10^{-2}$ . From (11), we obtain  $z_2^1 = 0.999194$ . The left hand side of (12) is equal to  $-4.67 \times 10^{-2}$  and  $\Delta U^2 = 4.36 \times 10^{-5} > 0$ . Moreover, individual 1 becomes worse off relative to the no destruction equilibrium, that is,  $\Delta U^1 + \Delta U^2 = -3.285 \times 10^{-3} < 0$ . The result is robust to different configurations of  $\gamma$  and  $\epsilon$ .

The intuition behind the welfare improvement is as follows. Individual 2 insures by transferring wealth from state  $b$  to state  $a$  (the bad state for her) and increases her date 0 consumption; she benefits from the higher price of asset 2 (the price of asset 1 cancels out from the date 0 budget constraint because  $z_1^2 = 1$  in equilibrium). Effectively, this is an economy with three states of the world. Individual 2 becomes better off by transferring wealth from state  $b$  to the other two states.

**Remark 6.** The incentives to destroy can be related to the literature on financial innovation: [Allen and Gale \(1994\)](#) and [Carvajal et al. \(2012\)](#), among others. We consider economies similar to the two-date set up of that literature. In particular, an entrepreneur (monopolist) owns two firms that pay identical dividends in every state of the world (two states for simplicity) and has preferences only for date 0 consumption. On the other hand, two types of investors, that do not own any asset, want to insure against future endowment risks. It is straightforward to construct examples where short sales are not allowed and it is optimal for the monopolist to destroy part of his output, that is, market value (date 0 consumption) is greater relative to the no destruction case. Destruction can be thought of as “innovation” in the sense that it “introduces” assets that can span more risks.

<sup>6</sup> [Hens and Pilgrim \(2002\)](#) give details.

**Remark 7.** Finally, the suboptimality of competitive allocations when risk sharing is restricted prompted Geanakoplos and Polemarchakis (1986) to define constrained suboptimality; and to demonstrate that public intervention that employs *instruments that do not augment risk-sharing opportunities* can implement Pareto improvements. The intervention, here, is effective precisely by augmenting insurance possibilities.

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