



## Myopia and monetary equilibria<sup>☆</sup>

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### ABSTRACT

In an infinitely lived, representative individual economy, important properties of competitive equilibria, such as determinacy and the non-existence of monetary equilibria, are not robust to the introduction of myopia. An individual is myopic if, at each date, he plans consumption only for that date and few periods that immediately follow; that is, his planning horizon,  $n$ , is finite. Equilibria with myopia can display real indeterminacy and allow for monetary as well as non-monetary steady states; thus, they share some of the features of equilibria in economies of overlapping generation. The equilibrium price dynamics (but not the consumption dynamics) of an exchange economy with extreme myopia,  $n = 1$ , are identical to the dynamics of an overlapping generation economy with two-period lives.

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## 1. Introduction

Two paradigms play an important role in macroeconomics and in monetary theory: following Ramsey (1928), the paradigm of an infinitely lived, representative individual (IL); and, following Samuelson (1958), the paradigm of overlapping generations (OG).

These two paradigms lead to starkly different conclusions for the determinacy and Pareto optimality of competitive equilibria as well as for the existence of monetary equilibria, where a fiat asset in positive net supply has positive value. As a consequence, the two paradigms have conflicting implications for the desirability and the effectiveness of fiscal and monetary policies. In OG economies, active policy may be necessary in order to attain efficiency; not so in the IL paradigm, where competitive equilibrium allocations are Pareto optimal. Also, in IL economies, competitive equilibria are (typically) locally unique or determinate: preferences, endowments and technology determine competitive allocations. As a consequence, it is possible to study the comparative statics of equilibria, which is necessary, for instance, in order to determine the lump-sum taxes that support particular Pareto optimal allocations. In OG economies, competitive equilibria need not be determinate: equilibrium allocations may depend not only on fundamentals, but also on (self-fulfilling) expectations of individuals regarding future prices. A fiscal transfer may not pin down a particular equilibrium allocation. Monetary equilibria are also an issue. In IL economies, assets with no intrinsic value, fiat money or aggregate nominal debt, cannot maintain a

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positive price at equilibrium. Quite to the contrary, OG economies allow for monetary equilibria and for monetary policy that has real effects.

That the two paradigms used in dynamic macroeconomics lead to divergent results and policy implications is an issue. Here, we introduce bounded rationality in an IL model, and we study the implications of the degree of bounded rationality on the determinacy and existence of monetary equilibria. We model bounded rationality in a simple manner: at each date, the representative individual plans short run consumption by maximizing a short run utility function and taking into account only short run prices and revenue. Though infinitely lived, the individual, at each date, plans consumption only for that date and  $n$  dates that follow. The degree of bounded rationality is the length,  $n$ , of the planning horizon of the individual. The limit case,  $n = \infty$ , corresponds to a fully rational representative individual and coincides with the standard IL economy. When  $n$  is finite, the individual is *myopic*. We show that in the presence of myopia the qualitative properties of the equilibrium dynamics resemble those of an OG economy. Namely, there exist one monetary steady state and one non-monetary steady state and the equilibrium is not necessarily locally unique. This result applies to pure exchange economies as well as to economies with production and capital accumulation. These findings suggest that the qualitative equilibrium properties of IL economies with fully rational individuals are not robust to the introduction of some form of myopia, whereas the qualitative properties of OG economies emerge in any IL myopic economy.

A myopic individual's consumption decision can be time inconsistent: actual consumption at date  $t$  may be different from what the individual had previously planned to consume at that date. This happens because at any date,  $t$  the individual plans his consumption taking into account elements that were not considered in the consumption plans computed previously, namely the prices, revenue and felicity at date  $t+n$ . And this time inconsistency of consumption plans can prevent the existence of an infinite sequence of prices that support a perfect foresight equilibrium. In fact, the market clearing price at date  $\tau$  given the consumption plan computed at date  $t < \tau$  can be different from the market clearing price given the actual consumption decision at date  $\tau$ . For this reason, we focus on "perfect foresight spot equilibria" (PFSE) that only require market clearing for spot markets at all dates and thus feasibility of the actual consumption path. In other words, differently from perfect foresight equilibria, PFSE do not require feasibility of all planned consumption paths, but only feasibility of the realized consumption path. When the representative individual is fully rational ( $n = \infty$ ), consumption decisions are time consistent and PFSE coincide with bona fide perfect foresight equilibria. In this instance, the equilibrium is unique.<sup>1</sup> Moreover, nominal assets in positive net supply have no value: time consistency implies that the value of actual consumption coincides with the value of planned consumption and hence cannot differ from the value of the representative individual's real resources. By contrast, when the representative individual is myopic, his actual consumption need not be equal to the consumption planned in the past. This weakens the link between the budget constraints and market equilibrium conditions and makes monetary equilibria and indeterminacy possible. This result is robust as it holds for any finite  $n$ , and it applies to exchange economies as well as to economies with production. In other words, optimality and determinacy of laissez-faire competitive equilibria are properties of the IL framework that are not robust to the introduction of myopia in the economy. This suggests that one could interpret the qualitative properties of OG economies (indeterminacy or the existence of monetary equilibria) as deriving from IL economies where individuals are myopic.

The most common setting in the OG literature is that of an exchange economy in which individuals live two periods. For this case, we show that any equilibrium price dynamics of a two-period life OG exchange economy that is invertible (in a sense that we define) can be replicated by an appropriate IL economy with extreme myopia, that is, with  $n = 1$ . Conversely, the equilibrium price dynamics of any given exchange IL myopic economy with extreme myopia, is identical to the equilibrium price dynamics of an appropriate OG exchange economy. This equivalence in price dynamics, evidently, does not carry over to the equilibrium dynamics of allocations: these are trivial in a representative agent exchange economy, whereas they can be rich in OG economies.

Ours is not the first paper that tries to link the IL and the OG paradigms. Aiyagari (1987, 1992) show that an IL economy can be obtained by introducing a bequest motive in the utility function of the individuals of an OG economy. Another approach starts from an IL economy proves that the introduction of cash in advance constraint, Huo (1987), or finance constraints, Woodford (1986), generate equilibrium dynamics equivalent to the dynamics of an OG economy with a two-period life span. Both approaches make the link between two extreme cases: the two-period life span OG economy and the IL economy. Neither obtains a link between IL economies and OG economies where individuals have life spans longer than two periods, which is the case that is empirically relevant. We start from IL economies and show how the qualitative properties of OG economies (even with relatively long lived individuals) can be reproduced by the introduction of some level of myopia. In addition, we show that the link between "short lived individual" OG exchange economies and extreme myopia IL exchange economies is tight for the equilibrium price dynamics, but it is not so as to what concerns the allocation dynamics.

The remainder of the paper is organized as follows: Section 2 describes the economy, the consumer behavior and two alternative hypothesis regarding the source of the production good: endowment or production technology involving capital and labor. Section 3 studies the property of the equilibrium price dynamics of the myopic economy. Section 4 compares myopia with OG. Section 5 concludes.

<sup>1</sup> Dana (1993) for exchange IL economies and Stokey and Lucas (1989) for IL economies with production.

## 2. The economy

There is one infinitely lived representative individual. Time is discrete,  $t=0, \dots$ , and there is one consumption good at each date. At the initial date,  $t=0$ , the representative individual is endowed with an amount,  $k_0 \geq 0$ , of capital and holdings,  $M$ , of a nominal asset. At any date  $t$ , he supplies inelastically one unit of labor in exchange for  $\omega_t$  units of the consumption good. His utility function is time separable and stationary,

$$U(c) = \sum_{t=0}^{\infty} \beta^t u(c_t), \quad 0 < \beta < 1,$$

where  $c = \{c_t\}_{t=0}^{\infty}$  is a non-negative consumption path and  $c_t$  is the level of consumption at date  $t$ .

As is standard, the cardinal utility index is strictly monotonically increasing,  $u' > 0$ , and strictly concave,  $u'' < 0$ , and

$$\lim_{c \rightarrow 0} u'(c) = +\infty.$$

The individual's rationality is bounded in the following sense: at any date,  $t$ , the individual is able to plan his consumption only for dates  $t, \dots, t+n$ , and, as a consequence, he solves the myopic optimization problem

$$\max_{\{c_t, \dots, c_{t+n}\}} \sum_{i=0}^{n-1} \beta^{t+i} u(c_{t+i}) + \delta \beta^{t+n} u(c_{t+n}), \quad \delta > 1, \tag{1}$$

$$\text{s.t.} \quad \sum_{i=0}^n p_{t+i} c_{t+i} \leq \sum_{i=0}^n p_{t+i} \omega_{t+i} + p_t r_t k_t + M_t. \tag{2}$$

The price of the consumption good is  $p_t$ , the real price of capital is  $r_t$ , and  $k_t$  and  $M_t$  are the holdings of capital and the nominal asset, respectively.

Three crucial assumptions are embedded in the myopic optimization problem. First, consumption plans can be time inconsistent in the sense that at some date,  $t$ , the planned consumption for a date  $t' \geq t$  does not coincide with what the individual planned at an earlier date,  $t' < t$  to be his date  $t'$  consumption. As the individual is boundedly rational, he is unaware of his time inconsistent behavior. In other words, at date  $t$ , the individual believes that his future self will behave exactly as he would then (that is at date  $t$ ) would like him to behave.<sup>2</sup> Second, the individual is cautious in the sense that, when he plans his consumption from date  $t$  to date  $t+n$ , he only takes into account the resources that are available from  $t$  to  $t+n$ . Third, the individual interprets his consumption at the end of his planning horizon,  $t+n$ , as a proxy for all future consumption, after  $t+n$ , that does not directly enter into his myopic maximization. This is captured by a factor  $\delta > 1$  that increases the weight that the individual attaches to felicity from consumption at the terminal date.<sup>3</sup>

The solution of the myopic optimization problem leads to the demand function that the individual plans at date  $t$  for commodities at dates  $t, \dots, t+n$ . For any  $i=0, \dots, n$ , the demand for the consumption good at date  $t+i$  is  $c_{i,t}$ . In particular,  $c_{0,t}$  denotes the demand for the date  $t$  good as expressed at date  $t$ . The individual consumption plan is time inconsistent if  $c_{i,t} \neq c_{0,t+i}$ , for some  $t$  and some  $i \in \{1, \dots, n\}$ .

First order conditions for the myopic optimization problem (1) and (2) imply that  $u'(c_{0,t})/p_t = \beta^i u'(c_{i,t})/p_{t+i}$ , for  $i=1, \dots, n-1$ , while  $u'(c_{0,t})/p_t = \delta \beta^n u'(c_{n,t})/p_{t+n}$ . Let  $\varphi$  denotes the inverse function of  $u'$ , then  $c_{i,t} = \varphi((p_{t+i} u'(c_{0,t})/p_t \beta^i)$ , for  $i=1, \dots, n-1$ , while  $c_{n,t} = \varphi((p_{t+n} u'(c_{0,t})/p_t \delta \beta^n)$ . Substituting these expressions in the budget constraint (2) and dividing by  $p_t$ ,

$$\sum_{i=0}^{n-1} \frac{p_{t+i}}{p_t} \left( \varphi \left( \frac{p_{t+i}}{p_t} \frac{u'(c_{0,t})}{\beta^i} \right) - \omega_{t+i} \right) + \frac{p_{t+n}}{p_t} \left( \varphi \left( \frac{p_{t+n}}{p_t} \frac{u'(c_{0,t})}{\delta \beta^n} \right) - \omega_{t+n} \right) = r_t k_t + \mu_t, \tag{3}$$

where  $\mu_t = M_t/p_t$  are real balances. This defines implicitly the individual's demand, at date  $t$ , for the consumption good then,  $c_{0,t}$ , as a function of prices and income in the interval of dates from  $t$  to  $t+n$  and of current real wealth  $r_t k_t + \mu_t$  at date  $t$ .

### 2.1. Production and exchange

There are two alternative specifications of production technologies of the consumption good: one corresponds to an exchange economy, and the other to an economy with production. Both technologies employ the unit of labor that is inelastically supplied.

<sup>2</sup> In the terminology of O'Donoghue and Rabin (2001), the individual is *naive*.

<sup>3</sup> There are many explanations for this myopic behavior. One possibility is that even if the individual has perfect foresight on the price level in the short run (from today to  $n$  periods after today), he has no idea of the long run level of prices (after  $n+1$  periods on) and therefore he cannot plan consumption that are too far in the future.

*Exchange economy:* At each date, the individual transforms the unit of labor into an amount  $\omega$  of the consumption good at the same date. Since labor is supplied inelastically, this is analogous to a situation where the individual receives a constant endowment  $\omega$  of the consumption good at each date. As there is no use for capital in the exchange economy,  $r_t = 0$ .

*Production economy:* At each date, the consumption good is produced in a competitive sector employing both labor and capital. The good can be consumed during the period or stored as an input for future production. Output per capita is a function of capital intensity,  $y_t = f(k_t)$  where  $f$  is a gross production function that includes the depreciated capital.

As is standard, the production function is smooth strictly increasing,  $f' > 0$ , and strictly concave,  $f'' < 0$ , and,

$$\begin{aligned} \lim_{k \rightarrow 0} f'(k) &= +\infty, & \lim_{k \rightarrow \infty} f'(k) &\in [0, 1), \\ \lim_{k \rightarrow 0} f(k) - kf'(k) &= 0, & \lim_{k \rightarrow \infty} f(k) - kf'(k) &= +\infty. \end{aligned}$$

In the production economy, capital at date  $t=0$  is needed to generate any positive consumption, and  $k_0 > 0$ .

## 2.2. Equilibrium concepts

In a perfect foresight equilibrium individuals anticipate correctly future prices and revenues.

For an exchange economy it suffices to consider the market for the consumption good, as the money market clears as a residual.

**Definition 1.** A perfect foresight equilibrium (PFE) of the exchange economy is a sequence of prices,  $\{p_i\}_{i=0}^{\infty}$ , such that, at every date  $t$ , spot and forward markets for goods are in equilibrium:

$$c_{i,t} = \omega, \quad i = 0, \dots, n.$$

Alternatively, for a production economy, equilibrium requires that, at every date, the aggregate excess demand for current and future consumption goods, nominal assets and capital is zero.

**Definition 2.** A perfect foresight equilibrium (PFE) of the production economy is a sequence of prices and rates of interest,  $\{p_i, \omega_t, r_i\}_{i=0}^{\infty}$ , such that at every date  $t$ , spot and forward markets for capital, labor and the nominal asset are in equilibrium:

$$r_t = f'(k_t), \quad \omega_t = f(k_t) - k_t f'(k_t), \quad p_t = f'(k_{t+1})p_{t+1}, \quad k_{t+i+1} = f(k_{t+i}) - c_{i,t} \quad i = 0, \dots, n.$$

These conditions follow from perfect competition in the capital, labor, and nominal asset markets and imply the absence of arbitrage between the capital and the nominal asset markets.

Note that, if  $n = \infty$ , the representative individual is not myopic, the standard IL models obtains and a PFE exists. In this instance, the real amount of nominal asset is zero. In an exchange economy, the unique equilibrium is characterized by the price dynamics  $p_{t+1} = \beta p_t$ . For a production economy, the same price dynamics sustains the stationary level of capital  $k_t = \hat{k}$ , where  $f'(\hat{k})\beta = 1$  is the unique interior steady state of the economy.

However, with effective myopia, a PFE, does not exist whenever  $\delta$  differs from 1.

**Proposition 1.** If  $n < \infty$ , and  $\delta \neq 1$ , then a PFE does not exist.

**Proof.** A PFE exists only if demand is time consistent. First order conditions require that  $u'(c_{n-1,t})/p_{t+n-1} = \delta \beta u'(c_{n,t})/p_{t+n}$ , while  $u'(c_{n-2,t+1})/p_{t+n-1} = \beta u'(c_{n-1,t+1})/p_{t+n}$ . Time consistency requires that  $c_{n-1,t} = c_{n-2,t+1}$  and  $c_{n,t} = c_{n-1,t+1}$ . This is a contradiction, since  $\delta \neq 1$ .  $\square$

A weaker definition of equilibrium is required.

**Definition 3.** A perfect foresight spot equilibrium (PFSE) for the exchange economy is a sequence of prices,  $\{p_i\}_{i=0}^{\infty}$ , such that, at any date  $t$ , the spot markets for good and the nominal asset are in equilibrium:

$$c_{0,t} = \omega.$$

**Definition 4.** A perfect foresight spot equilibrium (PFSE) for the production economy is a sequence of prices and rates of interest,  $\{p_i, \omega_t, r_i\}_{i=0}^{\infty}$ , such that, in every date  $t$ , the spot markets for capital labor and nominal asset are in equilibrium:

$$r_t = f'(k_t), \quad \omega_t = f(k_t) - k_t f'(k_t), \quad p_t = f'(k_{t+1})p_{t+1}, \quad k_{t+1} = f(k_t) - c_{0,t}.$$

At a PFSE, at each date, spot markets for the consumption good, nominal asset, capital and labor are in equilibrium. The representative individual has perfect foresight of future prices. But, market clearing is required only at the date production, consumption and the exchange of assets physically occur. A PFSE can be interpreted as a situation where forward markets do not exist, but individuals can borrow or lend for one period the nominal asset and capital. In this case, individuals plan

future consumption on the basis of their expectation about the spot price that will be observed in the future.<sup>4</sup> Evidently, a PFE is also PFSE but not conversely.

### 3. Equilibrium dynamics

A PFSE determines the dynamics of endogenous variables.

**Proposition 2.** For the exchange economy,

1. A PFSE is any sequence of prices,  $\{p_t\}_{t=0}^{\infty}$ , such that

$$M = \sum_{i=0}^{n-1} p_{t+i} \left( \varphi \left( \frac{p_{t+i} u'(\omega)}{p_t \beta^i} \right) - \omega \right) + p_{t+n} \left( \varphi \left( \frac{p_{t+n} u'(\omega)}{p_t \delta \beta^n} \right) - \omega \right). \tag{4}$$

2. (Monetary steady state) If  $M$  has the same sign as

$$\mu^{EE} := \sum_{i=0}^{n-1} \left( \varphi \left( \frac{u'(\omega)}{\beta^i} \right) - \omega \right) + \left( \varphi \left( \frac{u'(\omega)}{\delta \beta^n} \right) - \omega \right), \tag{5}$$

then a monetary steady state PFSE exists, such that  $p_t = M/\mu^{EE}$  and  $\mu_t = \mu^{EE}$ .

3. (Non-monetary equilibrium) If (real) balances are zero,  $M=0$ , then there exists a PFSE such that  $p_{t+1} = \rho p_t$ , with  $\rho > \beta$ .

4. (Indeterminacy) If  $M < 0$  and the monetary steady state exists, then the equilibrium is locally stable.

**Proof.**

1. At any date, the representative individual’s demand for current good is implicitly defined by Eq. (3). Considering that in the exchange economy,  $r_t k_t = 0$ , condition (4) follows by substituting  $c_{0,t} = \omega$  in (3) and multiplying both sides by  $p_t$ .

2. Evaluating the left hand side of (4) when prices are constant at level  $p^*$  yields the right hand side of (5) multiplied by  $p^*$ . If  $M$  has the same sign of  $\mu^{EE}$ , then (4) is satisfied for  $p^* = M/\mu^{EE}$ .

3. If  $M=0$  and  $p_{t+1} = \rho p_t$ , then dividing both sides of Eq. (4) by  $p_t$  yields

$$\sum_{i=0}^{n-1} \rho^i \left( \varphi \left( \frac{\rho^i u'(\omega)}{\beta^i} \right) - \omega \right) + \rho^n \left( \varphi \left( \frac{\rho^n u'(\omega)}{\delta \beta^n} \right) - \omega \right) = 0. \tag{6}$$

Since  $\delta > 1$ , while  $u'$  is a decreasing function and  $\varphi$  is its inverse, the left hand side of (6) is positive for  $\rho = \beta$  and negative for  $\rho = \beta \delta^{1/n} > \beta$ . Hence there exist  $\rho \in ]\beta, \beta \delta^{1/n}[$  that satisfies (6).

4. The indeterminacy of equilibrium follows from the local stability of the monetary steady state. The linearized price dynamics around the monetary steady state are such that there exists at least one eigenvalue with modulus smaller than one. This guarantees that, in addition to the monetary and the non-monetary steady states, there exist a continuum of monetary equilibria where prices converge to the monetary steady state. Let

$$G^{EE}(p_t, \dots, p_{t+n}) := \sum_{i=1}^{n-1} p_{t+i} \left( \varphi \left( \frac{u'(\omega) p_{t+i}}{\beta^i p_t} \right) - \omega \right) + p_{t+n} \left( \varphi \left( \frac{u'(\omega) p_{t+n}}{\delta \beta^n p_t} \right) - \omega \right) - M.$$

Then the dynamics of equilibrium price is implicitly defined by

$$G^{EE}(p_t, \dots, p_{t+n}) = 0.$$

Thus, the characteristic equation of the  $n$ -order dynamics of equilibrium price is

$$Q^{EE}(\lambda) := \sum_{i=0}^n G_i^{EE}(p_t, \dots, p_{t+n}) \lambda^i = 0,$$

<sup>4</sup> The concept of PFSE is similar to temporary equilibria studied by Grandmont (1977) with the difference that, here, individuals’ expectation about future prices are correct.

where  $G_i^{EE}$  is the derivative of  $G^{EE}$  with respect to price  $p_{t+i}$ . By a simple computation,

$$G_0^{EE}(p_t, \dots, p_{t+n}) = - \sum_{i=1}^n \frac{p_{t+i}}{p_t} \frac{u'(c_{i,t})}{u''(c_{i,t})},$$

$$G_i^{EE}(p_t, \dots, p_{t+n}) = c_{i,t} - \omega + \frac{u'(c_{i,t})}{u''(c_{i,t})}, \quad \text{for } i \geq 1.$$

At the monetary steady state, prices are equal to  $p^*$  and the characteristic equation becomes

$$Q^{EE*}(\lambda) = \sum_{i=1}^n \left( c_i^* - \omega + \frac{u'(c_i^*)}{u''(c_i^*)} \right) \lambda^i - \sum_{i=1}^n \frac{u'(c_i^*)}{u''(c_i^*)},$$

where  $c_i^*$  is  $c_{i,t}$  computed at the monetary steady state, that is, when  $p_t = p^*$ . Since  $Q^{EE*}(0) = -\sum_{i=1}^n (u'(c_i^*)/u''(c_i^*)) > 0$  and  $Q^{EE*}(1) = \sum_{i=1}^n (c_i^* - \omega) = \mu^{EE} < 0$ , there exists  $\lambda \in (0, 1)$  such that  $Q^{EE*}(\lambda) = 0$ .  $\square$

**Proposition 3.** For a production economy,

1. A PFSE is any sequence of capital, real balances and prices  $\{k_t, \mu_t, p_t, r_t, \omega_t\}_{t=0}^\infty$ , such that

$$\mu_t = -k_{t+1} + \sum_{i=1}^{n-1} \frac{1}{\prod_{j=1}^i f'(k_{t+j})} \left( \varphi \left( \frac{u'(f(k_t) - k_{t+1})}{\beta^i \prod_{j=1}^i f'(k_{t+j})} \right) - (f(k_{t+i}) - f'(k_{t+i})k_{t+i}) \right)$$

$$+ \frac{1}{\prod_{j=1}^n f'(k_{t+j})} \left( \varphi \left( \frac{u'(f(k_t) - k_{t+1})}{\delta \beta^n \prod_{j=1}^n f'(k_{t+j})} \right) - (f(k_{t+n}) - f'(k_{t+n})k_{t+n}) \right) \tag{7}$$

$$\mu_t = \frac{\mu_{t+1}}{f'(k_{t+1})}, \tag{8}$$

and  $\{p_t, r_t, \omega_t\}$  satisfy Definition 4.

2. (Monetary steady state) Let  $k^*$  be such that  $f(k^*) = 1$ , and let  $c^* := f(k^*) - k^*$ . If  $M$  has the same sign as

$$\mu^{PE} := \sum_{i=1}^{n-1} \left( \varphi \left( \frac{u'(c^*)}{\beta^i} \right) - c^* \right) + \left( \varphi \left( \frac{u'(c^*)}{\delta \beta^n} \right) - c^* \right) - k^*, \tag{9}$$

then a monetary steady state PFSE exists, such that  $k_t = k^*$  and  $\mu_t = \mu^{PE}$ .

- 3. (Non-monetary steady state) If  $\mu^{PE} > 0$  or  $\delta > u'(f(\hat{k}) - \hat{k})/u'(f(\hat{k}))$ , then a PFSE exists such that  $k_t$  is constant and the real amount of the nominal asset is zero. In this equilibrium  $p_{t+1}/p_t = \gamma > 0$ .
- 4. (Indeterminacy) If  $M > 0$  and the monetary steady state exists, then the equilibrium is locally stable.

**Proof.**

- 1. Eq. (8) follows from Definition 4, which also implies that  $p_{t+i}/p_t = 1/\prod_{j=1}^i f'(k_{t+j})$ ,  $c_{0,t} = f(k_t) - k_{t+1}$  and  $\omega_t = f(k_t) - k_t f'(k_t)$ . By substitution these expressions into (3), the dynamics of capital and of the real amount of nominal asset  $\mu_t$  are implicitly defined by the system of equations (7) and (8).

2. If  $(k, \mu)$  is a stationary state of the system of equations (7) and (8), then  $\mu = \mu/f(k)$ , which can be satisfied only if either  $f'(k) = 1$ , or  $\mu = 0$ . If  $f'(k) = 1$ , then  $k = k^*$  and  $p_t$  is stationary, then the right hand side of (7) equals the right hand side of (9). If, in addition,  $M$  has the same sign of  $\mu^{PE}$ , then it is possible to fix the stationary level of  $p_t$  at  $p^* = M/\mu^{PE}$ . By substituting  $k_t = k^*$  for all  $t$ , we obtain that  $\mu_t = \mu^{PE}$  and  $k_t = k^*$ , for all  $t$ , satisfies the system of equations (7) and (8).
3. By assumption, there exists  $\hat{k}$  such that  $\hat{k} = f(\hat{k})$  and  $f'(\hat{k}) > f'(\tilde{k})\tilde{k}$ . Thus, the right hand side of (7) is negative when  $k_t = \tilde{k}$  for all  $t$ . If (9) is positive, then the right hand side of (7) is positive when  $k_t = k^*$  for all  $t$ . If  $\delta > u'(f(\hat{k}) - \hat{k})/u'(f(\tilde{k}))$ , then the right hand side of (7) is positive when  $k_t = \hat{k}$  (i.e., for  $f'(\hat{k}) = 1/\beta$ ). Hence, there exists  $k < \hat{k}$  such that the left hand side of (7) is zero when  $k_t = k$  for all  $t$ . Thus,  $k_t = k$  and  $\mu_t = 0$  for all  $t$ , is a steady state equilibrium where  $p_{t+1}/p_t = 1/f'(k) > 0$ .
4. Indeterminacy obtains if the linearized dynamics around the monetary steady state has at least one eigenvalue with modulus smaller than 1. If  $H(k_t, \dots, k_{t+n})$  denotes the right hand side of (7), and

$$G^{PE}(k_t, \dots, k_{t+n+1}) := H(k_t, \dots, k_{t+n})f'(k_{t+1}) - H(k_{t+1}, \dots, k_{t+n+1}),$$

it follows from (8), that the dynamics of accumulation of capital is implicitly defined by  $G^{PE}(k_t, \dots, k_{t+n+1}) = 0$ . The characteristic equation is

$$Q^{PE}(\lambda) := \sum_{i=0}^{n+1} G_i^{PE}(k_t, \dots, k_{t+n+1}) \lambda^i = 0,$$

where  $G_i^{PE}$  is the partial derivative of  $G^{PE}$  with respect to  $k_{t+i}$ . Let

$$Q^{PE*}(\lambda) := Q^{PE}(\lambda) \Big|_{k_t = \dots = k_{t+n+1} = k^*}.$$

At the monetary steady state,

$$\begin{aligned} G_0^{PE}(k^*, \dots, k^*) &= H_1(k^*, \dots, k^*), \\ G_1^{PE}(k^*, \dots, k^*) &= f''(k^*)H(k^*, \dots, k^*) + H_2(k^*, \dots, k^*) - H_1(k^*, \dots, k^*), \\ G_i^{PE}(k^*, \dots, k^*) &= H_{i+1}(k^*, \dots, k^*) - H_i(k^*, \dots, k^*), \\ G_{n+1}^{PE}(k^*, \dots, k^*) &= -H_{n+1}(k^*, \dots, k^*), \end{aligned}$$

for  $2 \leq i \leq n$ , where  $H_i$  is the partial derivative of  $H$  with respect to its  $i$ th argument. At the monetary steady state,  $Q^{PE*}(1) = f'(k^*)H(k^*, \dots, k^*) = f'(k^*)\mu^*$  that is negative if  $M$  and the right hand side of (9) are positive. Thus, if  $Q^{PE*}(0) > 0$ , then  $Q^{PE*}(\lambda)$  has a zero between 0 and 1. Note that  $Q^{PE*}(0) = H_1(k^*, \dots, k^*)$ . Thus, consider

$$\begin{aligned} H_1(k_t, \dots, k_{t+1}) &= \sum_{i=1}^{n-1} \frac{1}{\prod_{j=1}^i f'(k_{t+j})} \left( \varphi' \left( \frac{u'(f(k_t) - k_{t+1})}{\beta^i \prod_{j=1}^i f'(k_{t+j})} \right) \frac{u''(f(k_t) - k_{t+1})f'(k_t)}{\beta^i \prod_{j=1}^i f'(k_{t+j})} \right) \\ &+ \frac{1}{\prod_{j=1}^n f'(k_{t+j})} \left( \varphi' \left( \frac{u'(f(k_t) - k_{t+1})}{\delta \beta^n \prod_{j=1}^n f'(k_{t+j})} \right) \frac{u''(f(k_t) - k_{t+1})f'(k_t)}{\delta \beta^n \prod_{j=1}^n f'(k_{t+j})} \right). \end{aligned}$$

Now, since  $f(k_t) - k_{t+1} = c_{0,t}$ ,  $\prod_{j=1}^i f'(k_{t+j}) = p_t/p_{t+i}$ , and for  $i < n$ ,  $p_{t+i}u'(c_{0,t}) = p_t\beta^i u'(c_{i,t})$ , it results

$$\varphi \left( \frac{u'(f(k_t) - k_{t+1})}{\beta^i \prod_{j=1}^i f'(k_{t+j})} \right) = c_{i,t}$$

and similarly,

$$\varphi \left( \frac{u'(f(k_t) - k_{t+1})}{\delta \beta^n \prod_{j=1}^n f'(k_{t+j})} \right) = c_{n,t}.$$

Let  $c^* := c_{0,t}$ ,  $c_i^* := c_{i,t}$ ,  $i > 0$ , when  $k_t = k_{t+1} = \dots = k_{t+n} = k^*$ . Then, as  $\varphi$  is the inverse function of  $u'$ ,

$$\varphi' \left( \frac{u'(f(k^*) - k^*)}{\beta^i f'(k^*)^i} \right) = \frac{1}{u''(c_i^*)} \quad \text{and} \quad \varphi' \left( \frac{u'(f(k^*) - k^*)}{\delta \beta^n f'(k^*)^n} \right) = \frac{1}{u''(c_n^*)}.$$

Thus, since  $f(k^*) = 1$ ,

$$Q^{PE*}(0) = H_1(k^*, \dots, k^*) = \sum_{i=1}^{n-1} \frac{u''(c_i^*)}{\beta^i u''(c_i^*)} + \frac{u''(c_n^*)}{\delta \beta^n u''(c_n^*)} > 0. \quad \square$$

In words, [Propositions 2 and 3](#) show that myopic economies display three remarkable features. First, both economies can display two steady states: the non-monetary steady state and the monetary steady state. In the non-monetary steady state, the real balances are zero and the prices change at a constant rate. In the monetary steady state, prices are constant and the sign of the nominal asset depends on the parameter  $\delta$  that affects the weight of the latest consumption in the myopic maximization problem. It is easy to verify that for  $\delta$  sufficiently large (small) the monetary steady state is compatible with fiat money (resp. aggregate debt). Second, the equilibrium dynamics is defined by a difference equation whose order depends on the degree of myopia  $n$ . For the Exchange Economy, the evolution of the equilibrium price follows a  $n$ -order difference equation. For the production economy, equilibrium capital accumulation path can be expressed with a difference equation of order  $n + 1$ . Third, the equilibrium can be indeterminate. Namely in the exchange economy (production economy) when the monetary steady state is consistent with aggregate debt (resp. fiat money), there exists a continuum of monetary equilibria that converge to this steady state.

There is intuition for each one of these features. The right hand sides of [\(4\)](#) and [\(7\)](#) represent the difference between, on the one hand, the value of the consumption plan from  $t$  to  $t+n$  and on the other hand, the value of the revenue perceived in those periods. Eqs. [\(4\)](#) and [\(7\)](#) state that this difference must equal the value of the real balances at time  $t$ . If actual consumption were equal to planned consumption, then in equilibrium the real balances could not differ from zero because of feasibility. However, myopia implies that in every new period the individual realizes he will have to consume for one additional future period and hence he will update his consumption plan accordingly. As a consequence, in equilibrium, his planned consumption need not be his actual consumption. In particular while the individual always plans to spend (or return) the real balances within the next  $n$  periods, he will never actually do so. This explains why, in the presence of myopia, real balances can differ from zero. When prices and capital are constant, equalities [\(4\)](#) and [\(7\)](#) translate into [\(5\)](#) and [\(8\)](#) identifying the monetary steady states of the exchange economy and the production economy, respectively. Still for a monetary steady state to exist it is also necessary that the sign of real balances matches the time preference of the individual. More precisely, with zero real balances and in the presence of constant prices, an impatient (patient) individual



current demand would be above (below) what is currently feasible. Equilibrium can be restored by providing the individual with negative (positive) real balances so that in every period current demand coincides with actual supply.

Further, in a PFSE the consumption plan must be such that the current demand for time  $t$  good equals its actual supply, that is  $\omega$  for the exchange economy and  $f(k_t) - k_{t+1}$  for the production economy. Since the markets for future consumption are not required to be in equilibrium today, there are many vectors of prices  $\{p_t, \dots, p_{t+n}\}$  and capital  $\{k_t, \dots, k_{t+n+1}\}$  satisfying this requirement. Thus, roughly speaking, the equilibrium dynamics for each economy could be determined by arbitrarily fixing all but the last elements of these vectors and finding the last element so that (4) and (7) are met. This also provides an intuition of why the equilibrium can be indeterminate. Nevertheless, it is not true that for any arbitrarily chosen vector of price  $\{p_t, \dots, p_{t+n-1}\}$  and capital  $\{k_t, \dots, k_{t+n}\}$  there exist an equilibrium  $p_{t+n}$  and  $k_{t+n+1}$  satisfying (4) and (7), respectively. In particular it could be that there is only one possible choice for the starting values of prices and capital, implying that the equilibrium is determinate. For this reason, in order to prove that indeterminacy is possible, we show that there is a continuum of initial possible choices for the initial condition each one generating a path that converges to the monetary steady state.

The dynamics of prices induced by a myopic representative individual can be rich in both economies. However, on the one hand, in the exchange economy the dynamics of allocation of resources is trivial as in every period the representative individual can only consume his constant endowment  $\omega$ . Thus, myopia can give reason for prices volatility in the presence of stationary consumption and production. On the other hand, the myopic production economy generates a non-trivial dynamics of capital and consumption. It strikes that the presence of myopia in the production economy allows to reach a level of steady state production and consumption that is not achievable when the representative individual is fully rational. More precisely, welfare in the myopic economy at the monetary steady state is higher than welfare in the unique interior steady state of the non-myopic economy. To see this point, note that because of the investment equations in Definitions 2 and 4, the level of consumption at a steady state is  $c = f(k) - k$  that is maximized for  $k = k^*$ . This maximum is achieved in the monetary steady state of the myopic economy but it is not sustainable at the unique interior steady state of the non-myopic economy that is  $\hat{k} < k^*$ .

To conclude, it is of interest to point out properties of the myopic economies steady states when myopia is low, that is for  $n$  large. To this purpose, a family of myopic economies is indexed by the degree of myopia  $n$ . For a given economy  $n$ , the real balances at the monetary steady state is  $\mu^*(n)$ . For the exchange economy,  $\rho(n)$  is the growth factor of prices at the non-monetary steady state.

#### Proposition 4.

1. At the monetary steady state, if  $n > -\ln(\delta)/\ln(\beta)$ , then  $\mu^*(n) < 0$ . Moreover,  $\lim_{n \rightarrow \infty} \mu^*(n) = -\infty$ .
2. At the non-monetary steady state of the exchange economy,  $\lim_{n \rightarrow \infty} \rho(n) = \beta$ .

#### Proof.

1. At the monetary steady state, the elements in the sum operators of expressions (5) and (9) are negative. If in addition, if  $n > -\ln(\delta)/\ln(\beta)$ , then  $(\varphi(u'(c)/\delta\beta^n) - c) < 0$  for any  $c > 0$ . Hence,  $\mu^*(n) < 0$ . Also, since  $(\varphi(u'(c)/\beta^i) - c)$  goes to  $-c$  as  $i$  increases, expressions (5) and (9) are unbounded for  $n$  that goes to infinity. Hence  $\lim_{n \rightarrow \infty} \mu^*(n) = -\infty$ .
2. At the non-monetary steady state of the exchange economy, the constant rate of growth of prices  $\rho(n)$  is  $\rho$  solving (6). For  $\delta > 1$ , the left hand side of (6) is positive for  $\rho = \beta$  and strictly negative for  $\rho = \beta\delta^{1/n}$ , hence  $\rho(n)$  is included between  $\beta$  and  $\beta\delta^{1/n}$ , implying  $\lim_{n \rightarrow \infty} \rho(n) = \beta$ .  $\square$

Proposition 4 suggests that when  $n$  is large but finite, the myopic economies maintain their two steady states provided the nominal asset has negative value. When  $n$  grows to infinity, the real balances at the monetary steady state explode and, in the exchange economy, the non-monetary steady state converges to the unique equilibrium of the non-myopic economy.<sup>5</sup>

## 4. Myopia and overlapping generations

In this section we compare the qualitative properties of the equilibrium dynamics of myopic economies with those of OG economies. While there are some striking similarities between the equilibrium price dynamics of these two families of economies, there is not an equivalence between OG economies and myopic economy for what regards the equilibrium dynamics of real variables such as consumption and capital.

Let us compare first a myopic exchange economy with OG exchange economy where individuals life span is  $l = (n+3)/2$ . Both economies equilibrium price dynamics is defined by a difference equation of order  $n$ . Both dynamics have two stationary states, one non-monetary steady state, where there is zero real balances and prices growth at a constant rate, and one

<sup>5</sup> The production economy does not necessarily have a non-monetary steady state when  $n$  is finite. Note however that the right hand side of (7) evaluated at  $k_t = \hat{k}$  for all  $t$  is  $\beta^n(\varphi(u'(f(\hat{k}) - \hat{k})/\delta\beta^n) - f(\hat{k}))$  that goes to 0 as  $n$  goes to infinity. Still, this is not enough to show that as  $n$  goes to infinity, a non-monetary steady state of the myopic production economy always converges to the steady state of the corresponding non-myopic economy.

monetary steady state with non-zero real balances constant prices. For both economies the sign of the real balances at the monetary steady state depends on the agents intertemporal preferences around the monetary steady state. Namely, if individuals discount future consumptions, the monetary steady state is consistent with negative real balance, whereas if individuals discount negatively future consumption, the monetary steady state is compatible with positive real balance.<sup>6</sup> In both economies the monetary steady state can be indeterminate. The comparison between OG economies with production and myopic production economies is more delicate. In fact, while myopia refers to any finite maximization horizon  $n$ , the literature on OG production economies focuses mainly on two-period life span individuals.<sup>7</sup> Still, there are some similarity between the equilibrium dynamics of an OG economy with production and the PFSE dynamics of a myopic production economy. First, both economies display a unique monetary steady state. Second, the sign of the real balances at the monetary steady state depends on the individual time preference in the same way as it happens for exchange economies. Third, if the monetary steady state is compatible with positive real balances, then a non-monetary steady state exists. Fourth, for a given production technology, the equilibrium level of capital at the monetary steady state is the same in OG and myopic economy. Fifth, the equilibrium can be indeterminate.

For what regards real variables dynamics however, myopia and OG are not alike. For instance in exchange economies, while the equilibrium dynamics of consumption of an OG economy can be rich, the dynamic of actual consumption (but not planned consumption) of a myopic exchange economy is trivial as, by construction, it must match the agent's constant endowment. For production economies, the equilibrium dynamics of consumption and capital can be rich in both myopic economies and a OG economies. However while for a given production technology, the two economies share the same level of capital at the monetary steady state, the equilibrium dynamics leading to these steady state are not alike.

#### 4.1. High level of myopia and OG

If the myopic individual maximization horizon is of two periods (that is,  $n = 1$ ), then the equilibrium price dynamics of a myopic exchange economy and that of an appropriate OG exchange economy are identical. Conversely, if the equilibrium price dynamic of an OG exchange economy where agents live two periods is invertible, then there exists a myopic exchange economy with  $n = 1$  that generates exactly the same price dynamics. In other words, the two models lead to the same set of equilibrium price dynamics even if consumption dynamics are always stationary for the myopic economy and not necessarily so for the OG economy.

In a myopic exchange economy with  $n = 1$ , the equilibrium (backward) dynamics of prices is homeomorphic to the equilibrium backward dynamics of the real balances  $\mu_t = M/p_t$ . In equilibrium the current and next period real balances satisfy

$$u'(\omega)\mu_t = \delta\beta u'(\omega + \mu_{t+1})\mu_{t+1}. \quad (10)$$

Now, consider an OG exchange economy where the aggregate amount of money is equal to  $M$ , individuals live two periods, they receive an endowment equal to  $e_0$  and  $e_1$  in their first and second period of life, respectively. We denote with  $x_t^j$  time- $j$ -consumption of the individual born in period  $i$ . The utility function of an individual born in  $t$  is  $v(x_t^t) + w(x_{t+1}^t)$ , where  $v$  and  $w$  are strictly increasing and concave and  $w$  satisfies Inada conditions. Let  $R_v(x) := -v''(x)x/v'(x)$ . We will refer to this kind of exchange economy as “standard two-period OG economy”. The equilibrium dynamics of the real balances  $\mu_t$  in this economy is implicitly defined by

$$v'(e_0 - \mu_t)\mu_t = w'(e_1 + \mu_{t+1})\mu_{t+1}. \quad (11)$$

The following proposition shows how to build a standard two-period OG economy whose set of equilibrium price dynamics coincides with that of any given myopic Exchange Economy with  $n = 1$ .

**Proposition 5.** *For any given myopic exchange economy when  $n = 1$  there exists a standard two-period OG economy whose set of equilibrium price dynamics coincides with the set of PFSE price dynamics of the myopic economy. In this OG economy the aggregate amount of money is equal to  $M$ , the utility function of individual born in  $t$  is  $ax_t^t + \delta\beta u(x_{t+1}^t)$ , with  $a = u'(\omega)$  and in the second period of life his endowment is  $\omega$ .*

**Proof.** Consider the myopic exchange economy when  $n = 1$ . In period  $t$ , the representative individual maximization problem is

$$\begin{aligned} \max_{\{c_t, c_{t+1}\}} & u(c_t) + \delta\beta u(c_{t+1}) \\ \text{s.t.} & p_t(c_t - \omega) + M_{t+1} \leq M_t, \\ & p_{t+1}(c_{t+1} - \omega) \leq M_{t+1}, \end{aligned}$$

<sup>6</sup> See also Aiyagari (1989) and Reichlin (1991) for OG economies.

<sup>7</sup> See for example Tirole (1985) and Jullien (1988).

with  $M_t$  given. It is straightforward to check that the PFSE price dynamics of the myopic economy is implicitly defined by

$$u'(\omega) = \frac{p_t}{p_{t+1}} \delta \beta u' \left( \omega + \frac{M}{p_{t+1}} \right). \tag{12}$$

Consider now the OG economy described in the proposition. The maximization problem of an individual born in period  $t$  is

$$\begin{aligned} \max_{x_t, x_{t+1}} \quad & a x_t^t + \delta \beta u(x_{t+1}^t) \\ \text{s.t.} \quad & p_t x_t^t + M_{t+1}^t \leq p_t e_0, \\ & p_{t+1} x_{t+1}^t \leq p_{t+1} e + M_{t+1}^t, \end{aligned}$$

where  $M_j^i$  is the quantity of nominal asset held at beginning of period  $j$  by the individual born in  $i$ . The first order condition leads to

$$a = \frac{p_t}{p_{t+1}} \delta \beta u'(x_{t+1}^t). \tag{13}$$

Assuming that at time 0 there is just one old individual that holds the total amount of nominal asset  $M$ , the equilibrium condition can be written as

$$x_t^t = e_0 - \frac{M}{p_t}, \tag{14}$$

$$x_{t+1}^t = e + \frac{M}{p_{t+1}}. \tag{15}$$

That means that young individuals consume their endowments minus the amount of good they sell in exchange for the nominal asset. Old individuals consume their endowments plus the quantity of good they buy with the nominal asset. Substituting the equilibrium conditions (14) and (15) in the first order condition (13) and considering the  $a = u'(\omega)$ , it follows that the equilibrium price dynamics of the OG economy is implicitly defined by (12). □

Proposition 5 shows that for  $n = 1$ , the equilibrium price dynamics of any myopic exchange economy can be replicated by the equilibrium price dynamics of an appropriate OG economy, however, the converse is not true. To see this point, note that expression (10) implies that myopia leads to equilibrium price dynamics that are invertible in the sense that there is at most one current equilibrium price for any given price in the following period. By contrast, some OG economies can generate price dynamics that are not invertible and hence cannot be reproduced with a myopic economy. Still, if a standard two-period OG exchange economy generates an equilibrium price dynamics that is invertible then the same price dynamics is generated by an appropriate myopic exchange economy.

**Proposition 6.** For any given standard two-period OG exchange economy satisfying  $R_v(x) < 1, \forall x$ , there exists an appropriate myopic exchange economy whose set of PFSE price dynamics coincides with that of the OG economy. Moreover, denoting with  $\mu_t = g(\mu_{t+1})$  the backward equilibrium dynamics of the real amount of nominal asset in the OG economy, it results that for the corresponding myopic economy  $u'(c) = g(c - \omega)/(c - \omega), \delta \beta = g'(0)$  and  $n = 1$ .

**Proof.** The following lemma provides the restriction on the equilibrium dynamics of the real balances in a myopic exchange economy with  $n = 1$ . □

**Lemma 1.** The difference equation  $\mu_t = g(\mu_{t+1})$ , with  $g : \mathbb{R} \rightarrow \mathbb{R}$  is the backward dynamics of a myopic economy with  $n = 1$  if and only if it satisfies the following properties:

1.  $g(0) = 0, g(m) < 0$  form  $< 0$  and  $g(m) > 0$  form  $> 0$ ;
2.  $g'(0) > 0$ ;
3.  $g'(m)m < g(m)$  form  $\neq 0$ .

Moreover, the representative individual's utility function of such myopic economy satisfies  $u'(c) = g(c - \omega)/(c - \omega) > 0$  and  $\delta \beta = g'(0)$ .

**Proof.** Necessary: Take any myopic exchange economy with  $n = 1$ . The equilibrium dynamics  $\mu_t$  is given by expression (10) that obviously satisfies conditions 1 and 2 with  $\partial \mu_t / \partial \mu_{t+1} |_{\mu_{t+1}=0} = \delta \beta$ . Differentiating expression (10) with respect to  $\mu_{t+1}$  and multiplying by  $\mu_{t+1}$ , condition 3 follows from  $u'' < 0$ . Thus, the equilibrium dynamics of any myopic economy with  $n = 1$  satisfies conditions 1–3.

Sufficient: Consider a function  $g$  satisfying 1–3 and fix  $\omega > 0$ . Take a myopic economy with  $n = 1$ , constant endowment  $\omega$  and individual's utility function such that  $u'(c) = g(c - \omega)/(c - \omega)$ , with  $u'(\omega)$  defined as  $\lim_{c \rightarrow \omega} g(c - \omega)/(c - \omega) = g'(0)$ , and  $\delta \beta = g'(0)$ . Condition 1 guarantees  $u' > 0$  while 3 guarantees  $u'' < 0$ , moreover,  $\delta \beta > 0$  for condition 2. Also, by substituting  $g(c - \omega)/(c - \omega)$  to  $u'(c)$  in expression (10), it results  $\mu_t = g(\mu_{t+1})$ .

In order to prove Proposition 6, we first show that  $g$ , the backward dynamics in the OG economy is well defined. Secondly, we prove that  $g$  satisfies conditions 1–3 of Lemma 1 so that a myopic economy whose equilibrium real balances dynamics is give by  $g$  exists.

Notice first that  $R_v(x) < 1$ ,  $\forall x$  implies  $\partial(v'(e_0 - \mu_t)\mu_t)/\partial\mu_t > 0$  for any  $\mu_t$ . Therefore, for a given  $\mu_{t+1}$  there is at most one  $\mu_t$  that satisfies Eq. (11) so that the relation  $\mu_t = g(\mu_{t+1})$  is well defined. Secondly we have to check that  $g$  satisfies conditions 1–3 of Lemma 1. Considering that  $\mu_t = g(\mu_{t+1})$  is implicitly defined by  $v'(e_0 - \mu_t)\mu_t - w'(e_1 + \mu_{t+1})\mu_{t+1} = 0$ , it is straightforward to check that  $g(0) = 0$  and that the sign of  $\mu_t$  is equal to the sign of  $\mu_{t+1}$  that is condition 1. By the implicit function theorem, it results

$$g'(\mu_{t+1}) = \frac{d\mu_t}{d\mu_{t+1}} = \frac{w'(e_1 + \mu_{t+1}) + \mu_{t+1}w''(e_1 + \mu_{t+1})}{v'(e_0 + \mu_t) - \mu_t v''(e_0 - \mu_t)}. \quad (16)$$

That is strictly positive for  $\mu_t = \mu_{t+1} = 0$  (condition 2). Finally,

$$g'(\mu_{t+1})\mu_{t+1} = \frac{w'(e_1 + \mu_{t+1})\mu_{t+1} + \mu_{t+1}^2 w''(e_1 + \mu_{t+1})}{v'(e_0 + \mu_t) - \mu_t v''(e_0 - \mu_t)}, \quad (17)$$

that is smaller than  $\mu_t$  as  $v''$  and  $w''$  are negative (condition 3).  $\square$

## 5. Conclusion

We have studied infinite long lived representative individual economies where the individual is myopic. Myopia implies that at beginning of each period the individual revises his consumption plan for the current and next  $n$  finite periods. We considered both a pure exchange economy and an economy with production and capital accumulation. We have shown that the presence of myopia in an IL economy allows for the existence of one monetary and possibly one non-monetary steady state and for indeterminacy of equilibrium. This implies that in IL economies, uniqueness of equilibrium and non-existence of monetary equilibria are not robust to the introduction of myopia. In other words, myopia in IL economies generates qualitative equilibrium properties that also characterize OG economies. Interestingly, in the myopic production economies the equilibrium levels of consumption and capital at the monetary steady state are strictly larger than those achievable at the steady state when the representative individual is rational ( $n = \infty$ ). For extreme level of myopia, ( $n = 1$ ), it is possible to construct myopic exchange economies and two-period life span OG economies that share the same set of equilibrium price dynamics even if they differ in the consumption equilibrium dynamics. Under this perspective, the model allows to move from the classical two-period life span OG exchange economy world into the IL world by changing the degree of myopia of the representative individual in the economy from  $n = 1$  to  $n = \infty$ , respectively.

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