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Sunspots, correlation and competition

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Abstract

Correlated equilibria in strategic market games played, simultaneously, by "overlapping generations" of players correspond to sunspot equilibria in the associated, competitive economy. We provide a necessary and sufficient condition for existence of effective correlation. An increase in the number of agents in a given economy may allow for effective correlation.

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1. Introduction

Extrinsic uncertainty has been extensively discussed in competitive equilibrium theory and macroeconomic theory since the pioneering contributions of Azariadis (1981) and Cass and Shell (1983). Sunspot equilibria represent endogenous stochastic fluctuations in market economies.

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Correlated equilibria for non-cooperative games, introduced by Aumann (1974, 1987) bear an evident similarity with sunspot equilibria for competitive markets and several have explored the possible connection between these two notions.[1]

Here, we illustrate the connection between correlated equilibrium and sunspot equilibrium in the simplest possible framework: an economy of overlapping generations in which individuals are strategic,[2] and, subsequently, we address the question that is our main concern: Does the degree of competition affect the possibility of endogenous fluctuations?

Our argument only illustrates the connection between correlated equilibrium and sunspot equilibrium: with finitely many (types of) individuals, strategic interactions do not permit that they coincide—they do in a large economy, as was demonstrated in Forges and Peck (1995). The construction serves to address our main concern, the link between the degree of competition and effective correlation or endogenous fluctuations.

The strategic behavior of individuals or the imperfectly competitive structure of markets we model as a market game, following Shapley (1976) or Shapley and Shubik (1977). The game is played simultaneously by a countable infinity of individuals or players, whose preferences and endowments parallel the structure of an economy of overlapping generations of Samuelson (1958)[3]; this eliminates considerations of dynamic or extensive form strategic choice and of refinements, such as, subgame perfection.

Forges and Peck (1995) first employed the simple model of overlapping generations to study correlated equilibria and introduced the construction of the correlated equilibrium that we use here.

Azariadis (1981) used the simple model of overlapping generations and a simple, two-state, stationary Markov process to show that the resulting equilibria may be subject to extrinsic uncertainty; and he gave necessary and sufficient conditions for the existence of a (non-trivial) two-state, stationary sunspot equilibrium. Here we construct a correlation

[1] Aumann et al. (1988) constructed an example of an imperfectly correlated or sunspot equilibrium of a market game with two types of individuals. Peck and Shell (1991) defined sunspot-Nash equilibria in market games with state-contingent security markets and proved that correlated equilibrium allocations are also sunspot-Nash equilibrium allocations with vanishing trades in securities. Forges (1991) considered extensive, rather than strategic form correlated equilibria (Forges, 1986, 1988), and proved that they coincide with sunspot equilibria for a market game with an atomless continuum of players and with complete security markets (Postlewaite and Schmeidler, 1978; Peck et al., 1992). Peck (1994) compared correlated and sunspot equilibria using the models of Azariadis (1981) and Cass and Shell (1983). Forges and Peck (1995) proved that the sunspot equilibria of a standard economy of overlapping generations and the correlated equilibria of the multi-stage market game, again, with an atomless continuum of players, that mimics it are equivalent. Maskin and Tirole (1987) considered a two-period, two-good, two-type model of an imperfectly competitive market and proved that extrinsic uncertainty can indeed affect the equilibrium, but only if there is imperfect correlation—neither perfect correlation nor complete independence. Davila (1999) considered correlated equilibria of a two-period, two-good, two-type economy as in Maskin and Tirole (1987) and sunspot equilibria of a market game played by the associated economy of overlapping generations and provided conditions under which the two coincide.

[2] Overlapping generations models with strategic agents have already been studied. Goenka et al. (1998) studied complex and chaotic equilibrium dynamics as in Grandmont (1985). Jacobsen (2000) considered imperfectly competitive cycles, while Heinemann (1997) discussed rationalizable sunspots in such a setup. Cordella and Datta (2002) characterized and compared competitive (Walras) and strategic (Cournot–Nash) stationary equilibria.

[3] The fundamental features of competitive equilibria in economies of overlapping generations derive from the double infinity of individuals and commodities and not from the temporal structure—Geanakoplos (1987) and Geanakoplos and Polemarchakis (1991).

device for this normal form game and then study the normal form, symmetric correlated equilibrium to demonstrate a natural parallel between stationary Markov correlated equilibria in a market game and stationary Markov equilibria with sunspots in the associated competitive economy.

Nash equilibria in a strategic market game converge to competitive equilibria of the associated economy—Dubey and Shubik (1978); this is the case in our set up as well (Proposition 1, Remark 1). Convergence fails in Cordella and Datta (2002), which is, still, a conundrum.

We provide a necessary and sufficient condition for existence of non-trivial correlated equilibrium (Theorem 1), analogous to the analysis in Azariadis (1981) and Azariadis and Guesnerie (1986) for sunspot equilibrium. We also show that an increase in the number of agents in a given economy may allow for effective correlation (Theorem 2).

A possible interpretation of the last result[4] is that in an economy with many agents, no agent has enough market power to influence the prices by much; so, even if the prices react to an "irrelevant" event, the agents are better off following that reaction. On the other hand, with few agents, each agent has enough market power to offset any "irrelevant" reactions of others, and that's what each agent prefers to do.

Further work should consider general settings.

2. The set up

Types of individuals are $t = 0, 1, \ldots,$ and individuals of each type are $i = 1, \ldots, n$; an individual is, thus, (i, t) or simply t, when only the type of the individual matters.

Commodities are $\tau = 1, \ldots$.

Individuals of type $t \neq 0$ are endowed with and derive utility from the consumption of commodities $\tau = t$ and $\tau = t + 1$.

The consumption of an individual is a bundle, (x^t, y^t), of commodities $\tau = t$ and $\tau = t + 1$, respectively.

The economy is stationary: an individual derives utility from consumption according to the (intertemporal) von Neumann–Morgenstern utility function

$$w(x, y) = u(x) + v(y), \quad (x, y) \geqslant 0,$$

and his endowment of commodities $\tau = t$ and $\tau = t + 1$ is

$$\omega = (e, f) \geqslant 0.$$

The cardinal indices, u and v are smooth and concave.

Individuals of type $t = 0$ are endowed with and derive utility from the consumption of commodity $\tau = 1$.

Such an economy, in which there are exactly n ($\geqslant 2$) individuals of each type, is denoted by E_n.

[4] We would like to thank an anonymous referee for suggesting this to us.

2.1. A strategic market game

Agents are (strategic) players in the game and can influence the prices of the commodities by their buy and sell orders.

The strategy set of an individual of type $t \neq 0$ is

$$\mathcal{S}^{i,t} = \{q_t^i\colon\ 0 \leqslant q_t^i \leqslant e\},$$

with the interpretation that q_t^i is the quantity of commodity $\tau = t$ that the individual offers in exchange for commodity $\tau = t + 1$.

Individuals of type $t = 0$ make no strategic choices.

A profile of strategies $\vec{q} = (q_t^i)$ determines the aggregate supply and, as a consequence, the price of every commodity, τ, respectively, according to

$$Q_\tau = \sum_i q_\tau^i, \quad \text{and} \quad p_\tau = \frac{1}{Q_\tau}.$$

The allocation of commodities to an individual of type $t \neq 0$ associated with a profile of strategies is

$$x^{i,t} = e - q_t^i, \qquad y^{i,t} = f + q_t^i \frac{p_t}{p_{t+1}},$$

and his utility or payoff is

$$w^{i,t} = u(x^{i,t}) + v(y^{i,t}) = u(e - q_t^i) + v\left(f + q_t^i \frac{p_t}{p_{t+1}}\right).$$

The aggregate consumption of individuals of type $t = 0$ is Q_1. It is important to note that the prices, p_t and p_{t+1} in the above expressions depend on the profile of strategies, $\vec{q} = (q_t^i)$; that is, to be exact, one should write $p_t(\vec{q}) = p_t(Q_t)$. This is the formulation of Shapley (1976) and Shapley and Shubik (1977).

2.2. The associated competitive economy

Prices of commodities are

$$p = (p_1, \ldots, p_\tau, \ldots) \geqslant 0.$$

The budget constraint of an individual of type $t \neq 0$ is

$$x^{i,t} = e - q_t^i, \qquad y^{i,t} = f + q_t^i \frac{p_t}{p_{t+1}}.$$

Here, the prices p_t and p_{t+1} are determined competitively, and individuals are price-takers. An individual demands or supplies commodities in order to maximize his utility subject to the budget constraint.

Individuals of type $t = 0$ consume Q_1, the aggregate supply of the commodity.

The interpretation and the analogy with the model of overlapping generations are evident: commodities are exchanged for fiat money, a medium of exchange, in fixed supply normalized to 1 per capita; exchange of commodities for balances occurs sequentially according to τ; the supply of balances complements the endowment of individuals of type $t = 0$.

2.3. Equilibria: Correlation and sunspots

A state of the world is $\vec{s} = (s_t; t \neq 0)$, with $s_t \in \{a, b\}$. The information partitions of individuals $t \neq 0$ are

$$\mathcal{P}^t = \left\{ P_a^t = \{\vec{s}\colon\ s_t = a\},\ P_b^t = \{\vec{s}\colon\ s_t = b\} \right\};$$

individuals $t = 0$ are uninformed, which is immaterial, since they make no strategic decisions.

Probability distributions over states of the world are generated by the stationary Markov transition probabilities

$$\begin{pmatrix} \pi(a|a) & \pi(b|a) \\ \pi(a|b) & \pi(b|b) \end{pmatrix} = \begin{pmatrix} \pi(a) & 1-\pi(a) \\ 1-\pi(b) & \pi(b) \end{pmatrix}.$$

With perfect serial correlation, $\pi(a|a) = \pi(b|b) = 1$ or $\pi(a) = \pi(b) = 1$.

With serial independence, $\pi(a|a) = \pi(a|b)$ and $\pi(b|b) = \pi(b|a)$ or $\pi(a) + \pi(b) = 1$.

This describes the correlation device.

A strategy of an individual, $t \neq 0$, is a pair $\{q_t(a), q_t(b)\}$: the quantity supplied by the individual as a function of the information he receives.

A symmetric, stationary, Markov, correlated equilibrium for the strategic market game is described by transition probabilities and strategies, $\{\pi(a), \pi(b), q(a), q(b)\}$, such that

$$q(a) = \arg\max_q \mathrm{E}_{s'|a} u(e-q) + v\left(f + q\frac{p(q+(n-1)q(a))}{p(s')}\right), \quad s' \in \{a, b\},$$

and

$$q(b) = \arg\max_q \mathrm{E}_{s'|b} u(e-q) + v\left(f + q\frac{p(q+(n-1)q(b))}{p(s')}\right), \quad s' \in \{a, b\}.$$

Recall that the price, $p(.)$, in the above expressions depend on the profile of strategies. Note that the stationary structure allows us to formulate the above.

This is a symmetric, stationary, Markov Nash equilibrium of the game extended by the correlation device, following the concept introduced by Aumann (1974, 1987).

For the associated economy, a stationary, Markov, competitive equilibrium with sunspots is described by transition probabilities, prices of commodities and quantities supplied, $\{\pi(a), \pi(b), p(a), p(b), q(a), q(b)\}$, such that

$$q(s) = \arg\max_q\ \mathrm{E}_{s'|s}\ u(e-q) + v\left(f + q\frac{p(s)}{p(s')}\right), \quad s, s' \in \{a, b\}.$$

This is the formulation of Azariadis (1981).

3. Results

We restrict attention to interior values: $0 < q(a) \leqslant q(b) < e$, as standard restrictions on preferences and endowments allow; also, to non-degenerate probability distributions: $0 < \pi(a), \pi(b) < 1$.

Proposition 1. *A correlated equilibrium for the strategic market game,* $\{\pi(a), \pi(b), q(a), q(b)\}$*, is a solution to the system of equations*:

$$u'\big(e - q(a)\big) = \frac{n-1}{n}\left(\pi(a)v'\big(f + q(a)\big) + \big(1 - \pi(a)\big)v'\big(f + q(b)\big)\frac{q(b)}{q(a)}\right),$$
$$u'\big(e - q(b)\big) = \frac{n-1}{n}\left(\pi(b)v'\big(f + q(b)\big) + \big(1 - \pi(b)\big)v'\big(f + q(a)\big)\frac{q(a)}{q(b)}\right),$$

with

$$p(a) = \frac{1}{q(a)} \quad \text{and} \quad p(b) = \frac{1}{q(b)}.$$

A sunspot equilibrium for the associated competitive economy, $\{\pi(a), \pi(b), p(a), p(b), q(a), q(b)\}$*, is a solution to the system of equations*:

$$u'\big(e - q(a)\big) = \pi(a)v'\big(f + q(a)\big) + \big(1 - \pi(a)\big)v'\big(f + q(b)\big)\frac{q(b)}{q(a)},$$
$$u'\big(e - q(b)\big) = \pi(b)v'\big(f + q(b)\big) + \big(1 - \pi(b)\big)v'\big(f + q(a)\big)\frac{q(a)}{q(b)}.$$

Remark 1. Proposition 1 shows that as the number of individuals in each type or generation, n, increases, correlated equilibria of the strategic market game coincide with sunspot equilibria of the associated economy. This is an application of the argument in Dubey and Shubik (1978), without reference to no-trade equilibria that arise naturally in market games irrespective of the size of the market. The identification of correlated equilibria with sunspot equilibria of the associated competitive economy in the case of atomless economies of overlapping generation was demonstrated in Forges and Peck (1995).

Following Cass and Shell (1983), *extrinsic uncertainty is effective* or sunspots do matter, at a competitive equilibrium, if $q(a) \neq q(b)$; extrinsic uncertainty is effective if it is effective at some competitive equilibrium. Similarly, we shall say that *correlation is effective,* at a correlated equilibrium, if $q(a) \neq q(b)$; correlation is effective if it is effective at some correlated equilibrium.

Remark 2. As it is clear from Proposition 1, the degree of competition, $(n-1)/n$, acts as a discount factor for second period consumption, and, as we shall note in the following analysis, it affects the possibility of effective correlation.

A correlated equilibrium where $q(a) = q(b) = q_n^*$ and, as a consequence, correlation is ineffective, exists: it obtains as the solution to the equation:

$$u'(e - q) = \frac{n-1}{n} v'(f + q),$$

and it is the symmetric, stationary, interior Nash equilibrium in pure strategies of the normal form market game in discussion. Similarly, a competitive equilibrium where

$q(a) = q(b) = q^*$ and, as a consequence, sunspots are ineffective, exists: it obtains as the solution to the equation:

$$u'(e-q) = v'(f+q),$$

and it is the stationary equilibrium of the associated competitive economy.

The existence of correlated equilibria with effective correlation depends on the curvature of the cardinal utility indices of individuals; equivalently, the slope of the supply of savings with respect to the rate of interest or the relative risk aversion of individuals over (net) second period consumption: if $v''(f+q)q + v'(f+q) < 0$ (> 0), then, savings is an decreasing (increasing) function of the interest factor, p_t/p_{t+1}, and relative risk aversion over (net) second period consumption is higher (lower) than 1. Simple characterizations obtain if the sign of the expression $v''(f+q)q + v'(f+q)$ does not vary with $q > 0$; the argument is as in Azariadis (1981).

Proposition 2. *If $v''(f+q)q + v'(f+q) > 0$, $0 < q < e$, then correlation is ineffective.*

Proposition 3. *If $v''(f+q)q + v'(f+q) < 0$, $0 < q(a) \leqslant q \leqslant q(b) < e$, then, at a correlated equilibrium with $\pi(a) + \pi(b) \geqslant 1$, correlation is ineffective.*

Remark 3. Signals are serially independent if $\pi(a) + \pi(b) = 1$; under the conditions of Proposition 3, independence does not allow for effective correlation.

Following the spirit of the above propositions, we now find a necessary and sufficient condition for effective correlation. Let $D = \{q\colon\ v''(f+q)q + v'(f+q) < 0\} \subseteq (0, e)$.

Theorem 1. *Correlation is effective if and only if $q_n^* \in D$, and*

$$-\frac{d}{dq}v'\big(f+q_n^*\big)q_n^* > \frac{n}{n-1}\frac{d}{dq}u'\big(e-q_n^*\big)q_n^*$$

or, equivalently,

$$-\big(v''\big(f+q_n^*\big)q_n^* + v'\big(f+q_n^*\big)\big) > -\frac{n}{n-1}\big(u''\big(e-q_n^*\big)q_n^* - u'\big(e-q_n^*\big)\big).$$

Our second result is that the degree of competition, $(n-1)/n$, enhances the possibility of effective correlation.

Theorem 2. *If correlation is effective for some $\bar{n}$, then correlation is effective for all economies with $n \geqslant \bar{n}$.*

The following parametric example illustrates our results.

Example. The utility function of individuals is $w(x, y) = u(x) + v(y) = -x^{-k} - y^{-2k}$, where k is a natural number, and their endowment is $\omega = (e, f) = (10, 2)$. It is easy to check that the condition $v''(f+q)q + v'(f+q) < 0$ is satisfied for $q > 1/k$, that is, here, $D = (1/k, 10)$.

First, we fix a value of k and we show that $q(a)=1$ and $q(b)=2$ obtain at equilibrium for some n. The condition,

$$v'\big(f+q(a)\big)q(a) > \frac{n}{n-1}u'\big(e-q(b)\big)q(b)$$

(see Lemma 2 in Appendix), translates to

$$\frac{8}{3}\left(\frac{8}{9}\right)^k > \frac{n}{n-1}$$

for these values. Since $(8/3)(8/9)^k$ converges to 0 as k increases and is less than 1 for $k \geqslant 9$, for any fixed value of k, between 1 and 8, there exists some $\bar{n}$ such that correlation is effective with values $q(a)=1$ and $q(b)=2$ for all economies with $n \geqslant \bar{n}$.

Also, for any fixed $\bar{n}$, there exists a k, between 1 and 8, such that $q(a)=1$ and $q(b)=2$ constitute an equilibrium for all economies with $n \geqslant \bar{n}$.

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Appendix

Proof of Proposition 1. For the strategic market game, the result follows from the definition of a correlated equilibrium and the necessary and sufficient first order conditions for strategies to be mutual optimal responses. For the associated competitive economy, the result follows from the definition of an equilibrium and the necessary and sufficient first order conditions for utility maximization subject to the budget constraint. □

Claim 1. *At a correlated equilibrium,*

$$\pi(a) = \frac{\frac{n}{n-1}u'(e-q(a))q(a) - v'(f+q(b))q(b)}{v'(f+q(a))q(a) - v'(f+q(b))q(b)},$$

$$\pi(b) = \frac{\frac{n}{n-1}u'(e-q(b))q(b) - v'(f+q(a))q(a)}{v'(f+q(b))q(b) - v'(f+q(a))q(a)}.$$

while, at the corresponding sunspot equilibrium,

$$\pi(a) = \frac{u'(e-q(a))q(a) - v'(f+q(b))q(b)}{v'(f+q(a))q(a) - v'(f+q(b))q(b)},$$

$$\pi(b) = \frac{u'(e-q(b))q(b) - v'(f+q(a))q(a)}{v'(f+q(b))q(b) - v'(f+q(a))q(a)}.$$

Proof. Follows directly from Proposition 1. □

Proof of Proposition 2. By the hypothesis, the function $v'(f+q)q$ is strictly monotonically increasing, as is, by concavity, the function $u'(e-q)q$. We argue by contradiction. Take at a correlated equilibrium, $q(a) < q(b)$, and, as a consequence,

$$v'\big(f+q(a)\big)q(a) < v'\big(f+q(b)\big)q(b).$$

Since $\pi(a) < 1$, from the expression for $\pi(a)$ in Claim 1,

$$\frac{n}{n-1}u'\big(e-q(a)\big) > v'\big(f+q(a)\big),$$

and, as a consequence, $q(a) > q_n^*$. Similarly, since $\pi(b) < 1$, $q(b) < q_n^*$, it follows that $q(b) < q(a)$, a contradiction. □

Proof of Proposition 3. By the hypothesis, the function $v'(f+q)q$ is strictly monotonically decreasing, while, from concavity, the function $u'(e-q)q$ is monotonically increasing. We suppose that $\pi(a)+\pi(b) \geqslant 1$, and argue by contradiction. Take at a correlated equilibrium, $q(a) < q(b)$, and, as a consequence, $v'(f+q(a))q(a) > v'(f+q(b))q(b)$. With $\pi(a) \geqslant (1-\pi(b))$, it follows from the expressions for $\pi(a)$ and $\pi(b)$ in Claim 1, at a correlated equilibrium, $u'(e-q(a)) \geqslant u'(e-q(b))$, and, therefore, $q(a) \geqslant q(b)$, a contradiction. □

Proof of Theorem 1. The proof is based on two lemmas. Lemma 1 provides a necessary, but not sufficient, condition for effective correlation while Lemma 2 presents a sufficient condition following the argument in the proof of Lemma 1.

Lemma 1. *Correlation is effective only if there exists a $\hat{q} \in D$, such that*

$$-\frac{d}{dq}v'(f+\hat{q})\hat{q} > \frac{n}{n-1}\frac{d}{dq}u'(e-\hat{q})\hat{q}$$

or, equivalently,

$$-\big(v''(f+\hat{q})\hat{q}+v'(f+\hat{q})\big) > -\frac{n}{n-1}\big(u''(e-\hat{q})\hat{q}-u'(e-\hat{q})\big).$$

Proof. If correlation is effective at $\{\pi(a),\pi(b),q(a),q(b)\}$ and $q(a) < q(b)$, then

$$v'\big(f+q(a)\big)q(a) > v'\big(f+q(b)\big)q(b).$$

But, from the expressions for $\pi(a)$ and $\pi(b)$ in Claim 1, $0 < \pi(a), \pi(b) < 1$ if and only if

$$v'\big(f+q(b)\big)q(b) < \frac{n}{n-1}u'\big(e-q(a)\big)q(a) < v'\big(f+q(a)\big)q(a),$$
$$v'\big(f+q(b)\big)q(b) < \frac{n}{n-1}u'\big(e-q(b)\big)q(b) < v'\big(f+q(a)\big)q(a)$$

or, equivalently,

$$v'\big(f+q(b)\big)q(b) < \frac{n}{n-1}u'\big(e-q(a)\big)q(a),$$

$$\frac{n}{n-1}u'\big(e-q(b)\big)q(b) < v'\big(f+q(a)\big)q(a)$$

or, equivalently,

$$v'\big(f+q(a)\big)q(a) > \frac{n}{n-1}u'\big(e-q(b)\big)q(b),$$
$$v'\big(f+q(b)\big)q(b) < \frac{n}{n-1}u'\big(e-q(a)\big)q(a).$$

These inequalities can be satisfied simultaneously only if the function $v'(f+q)q$ decreases faster than the function $\frac{n}{n-1}u'(e-q)q$ increases somewhere in D. □

Lemma 2. *If there exist*

(1) $\hat{q} \in D$, *such that*

$$-\frac{d}{dq}v'(f+\hat{q})\hat{q} > \frac{n}{n-1}\frac{d}{dq}u'(e-\hat{q})\hat{q}$$

or, equivalently,

$$-\big(v''(f+\hat{q})\hat{q}+v'(f+\hat{q})\big) > -\frac{n}{n-1}\big(u''(e-\hat{q})\hat{q}-u'(e-\hat{q})\big),$$

and

(2) $\underline{q}, \bar{q} \in D$, *with* $\underline{q} < \hat{q} < \bar{q} \in D$, *such that*

$$v'(f+\underline{q})\underline{q} > \frac{n}{n-1}u'(e-\bar{q})\bar{q},$$

then correlation is effective.

Proof. Following the proof of Lemma 1, at an effective correlated equilibrium, $\{\pi(a), \pi(b), q(a), q(b)\}$ with $q(a) < q(b)$, $0 < \pi(a), \pi(b) < 1$ if and only if the following inequalities are satisfied

$$v'\big(f+q(a)\big)q(a) > \frac{n}{n-1}u'\big(e-q(b)\big)q(b),$$
$$v'\big(f+q(b)\big)q(b) < \frac{n}{n-1}u'\big(e-q(a)\big)q(a).$$

It now suffices to set $q(a)=\underline{q}$ and $q(b)=\bar{q}$. □

The only if part of the theorem follows directly from Lemma 1. To prove the if part of the theorem, recall that at q_n^*, $v'(f+q)=\frac{n}{n-1}u'(e-q)$. Also note that the function $u'(e-q)q$ is strictly monotonically increasing by concavity while the function $v'(f+q)q$ is strictly monotonically decreasing for $q \in D$. Now, by the hypothesis of the theorem one can find $\underline{q}$ and $\bar{q}$, $\underline{q} < q_n^* < \bar{q} \in D$, such that we have $v'(f+\underline{q})\underline{q} > \frac{n}{n-1}u'(e-\bar{q})\bar{q}$. The if part now directly follows from Lemma 2. □

Proof of Theorem 2. As n increases, $(n-1)/n$ increases or, equivalently, $n/(n-1)$ decreases to 1. If the hypotheses (in Lemmas 1, 2 and Theorem 1) are satisfied for $\bar{n}$, then they are satisfied for all $n \geqslant \bar{n}$. □

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