

The Identification of Preferences from Equilibrium Prices under Uncertainty¹

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1. INTRODUCTION

Explanation and prediction require the behavior of individuals, which is observable, to identify their characteristics, which are not.

The preferences or utility functions of individuals are unobservable—beliefs are unobservable as well, even though they may not be exogenous and might vary with equilibrium prices or aggregate behavior.

In a market, it is competitive equilibria that are observable. The theory fails to specify out of equilibrium behavior and, as a consequence, demand

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at arbitrary prices and incomes is not observable. Experimental observations may be less restrictive.

Alternatively, for demand out of equilibrium, observations may involve different degrees of aggregation. At the most disaggregated level, one can observe the demand of individuals as prices and the allocation of endowments or revenue varies. The endowments of individuals may be in part unobservable, though redistributions of revenue are mostly observable; production possibilities may be observable or not.

The competitive equilibrium correspondence, which associates equilibrium prices of commodities and assets with allocations of endowments, identifies the cardinal utility functions and beliefs of individuals; this is the case even if the asset market is incomplete. Alternatively, the aggregate demand for commodities and assets identifies the cardinal utility functions and beliefs of individuals.

It follows that generality in competitive equilibrium theory as put forward by Arrow and Debreu [2] and McKenzie [31], for abstract economies, and by Radner [35], for economies with an operative asset market, allows for explanation and prediction.

In Chiappori *et al.* [10], the competitive equilibrium correspondence identifies the ordinal utility functions of individuals under certainty. The argument builds on earlier arguments of Brown and Matzkin [4].

Under certainty, the transfer paradox in Leontief [28], generalized in Donsimoni and Polemarchakis [16], makes it clear that knowledge of the ordinal utility functions is necessary in order to assess the welfare effects of transfers.

Under uncertainty, with an incomplete asset market, the identification of preferences from observed behavior has strong positive as well as normative implications. A competitive equilibrium allocation is not optimal; more pertinently, according to Geanakoplos and Polemarchakis [20], redistributions of portfolios of assets can result in a Pareto improvement; alternatively, following Herings and Polemarchakis [26], the regulation of prices and the imposition of rationing in order to attain market clearing or, following Citanna *et al.* [11], the taxation of assets can be Pareto improving. The question of the informational requirements for the determination of improving redistributions of portfolios of assets, regulation of prices, or tax rates immediately arises.

The identification of preferences from the equilibrium correspondence allows possibly counterintuitive distributional effects of financial innovation, as in Hart [25], to be predicted without ambiguity; alternatively, it allows for the determination of Pareto improving financial innovation, as in Cass and Citanna [6] or Elul [19]. In economies with production, it allows for the determination of the investment decisions of firms as in Drèze [17] without recourse to unobservable characteristics.

The argument for identification is local. Given any profile of endowments with equilibrium prices, we uniquely recover the associated consumption allocation as well as preferences over consumption in a neighborhood of this allocation. This argument extends immediately to preferences over the whole consumption set if additional assumptions on preferences assure that the associated allocations are attained at some equilibrium.

The introduction of production possibilities poses no problem if the production choices of firms are observable. However, "household production" prevents identification even if individual demand, which aggregates consumption and production decisions, is observable.

The lack of available data and problems with econometric estimation procedures notwithstanding, it is an important theoretical question whether the necessary information concerning the unobservable characteristics of individuals can be identified from their observable, market behavior.

Under certainty, the identification of the preference relation of an individual from his demand behavior as the prices of commodities and his income vary is evident, under standard regularity assumptions; sufficient conditions for identification were examined in detail in Mas-Colell [30]. The identification of the preference relations of individuals is less evident if only aggregate demand behavior is observable; it is not evident that it is impossible to disaggregate the observed behavior into different families of individual demand functions generated by different profiles of utilities; indeed, an example with quasilinear preferences shows that this may be the case. Nevertheless, under a condition of nonvanishing income effects, the Slutsky decomposition of the demand functions of individuals can be exploited to identify their preferences from aggregate behavior; this was the argument in Chiappori *et al.* [10]. Indeed, the equilibrium correspondence, a priori less informative than aggregate demand, suffices for the identification of the preferences of individuals; which is surprising, since, on the equilibrium correspondence, prices and endowments do not vary independently, and there are, typically, finitely many prices associated with each profile of endowments. Identification from the equilibrium correspondence requires variation in the allocation of endowments. Nevertheless, observation of the distribution of income associated with alternative allocations of endowments and associated competitive equilibrium prices suffices for identification.

Under uncertainty, with an operative, possibly incomplete asset market, the identification of preferences even from individual demand behavior, though possible, is not evident: the first-order conditions for individual optimization do not determine, at least immediately, the marginal rate of substitution between consumption at different states of the world.

With multiple commodities, the variation of relative price of commodities at each state of the world permits identification when preferences are

state-separable, as in Geanakoplos and Polemarchakis [22]. This is the case even if the endowment of the individual in commodities is held fixed; a different and simpler argument is possible if the endowment or income of the individual in commodities at each state of the world can vary.

The argument for identification extends to the more restrictive settings where only aggregate demand as a function of individual first-period incomes and all prices is observable. This implies directly that the equilibrium correspondence identifies preferences when they are state-separable and when there are several goods traded at each state.

In the limiting case of one commodity, identification from aggregate asset demand is in general not possible: Dybvig and Polemarchakis [18] and McLennan [32] give examples where identification fails, while restrictions on the structure of payoffs of assets, as in Dybvig and Polemarchakis [18] or Polemarchakis and Rose [34], or on the utility function that represents the preferences, as in Green *et al.* [24], permit identification. Nevertheless, since along the equilibrium correspondence individual asset demand and hence consumption can be recovered, the Hessian of the utility function can be recovered.

The identification of beliefs poses additional problems in the limiting case of one commodity as in Polemarchakis [33]. With multiple commodities the identification of beliefs is obtained without further complications.

The identification of unobservable characteristics is distinct from the integrability of demand functions or from the restrictions that apply to individual or aggregate demand or the equilibrium correspondence.

Integrability, introduced by Samuelson [36] and further considered in Debreu [12, 13], concerns conditions for the existence of a complete and transitive preference relation that generates demand behavior; identification presupposes the existence of an underlying preference relation, often one that satisfies additional regularity conditions.

In principal, restrictions on observables may fail to arise even if identification is possible. Nevertheless there are a number of caveats, which may account for some confusion on the matter.

For aggregate excess demand under certainty, with variations restricted to the relative prices of commodities, restrictions vanish; Debreu [14], Mantel [29], and Sonnenschein [38, 39] developed a global argument for economies with a sufficient number of individuals relative to the number of commodities, Geanakoplos and Polemarchakis [21] characterized aggregate excess demand at a point, while Chiappori and Ekeland [7] gave a complete local characterization of aggregate excess demand. Without variation in the endowments or incomes of individuals, identification is not possible either. The failure of restrictions on aggregate excess demand extends to economies with uncertainty and an incomplete asset market, as in Bottazzi and Hens [3], Chiappori and Ekeland [8, 9], or Gottardi and Mas-Colell [23].

Since the argument for identification does not use all restrictions utility maximization imposes on individual demand, global restrictions on the equilibrium correspondence remain. The point was made for the case of certainty by Brown and Matzkin [5] and was further developed in Chiappori *et al.* [10] and DeMichelis *et al.* [15].

In general, identification implies that restrictions on unobservables have observable implications. Indeed, recoverability implies that all additional restrictions on preferences or beliefs translate immediately to restrictions on the equilibrium correspondence.

Concerning further research, in a more applicable argument, one needs to take into account that for time series data, prices and incomes might be part of one, intertemporal equilibrium, and not points on an equilibrium correspondence. According to Kübler [27], the assumption of time separable expected utility restores global restrictions in an intertemporal model. For identification, the assumption of separable utility is not enough, since sufficiently complete asset markets allow individuals to smooth their expenditure across dates and states of the world. Also, optimizing individuals take all prices and dividends into account when choosing their portfolio and consumption plans. However, observations can only consist of one sample path, and it seems unlikely that identification of preferences is possible without any knowledge of equilibrium prices at nodes that do not lie on the sample path.

2. THE ECONOMY

Individuals are $i \in \mathcal{I} = \{1, \dots, I\}$, a finite, nonempty set.

States of the world, exhaustive and exclusive descriptions of the environment, are $s \in \mathcal{S} = \{1, \dots, S\}$, a finite, nonempty set.

Commodities are $l \in \mathcal{L} = \{1, \dots, L\}$, a finite, nonempty set.

At the state of the world s , commodities are $(l, s) \in \mathcal{L} \times \{s\}$, and a bundle of commodities is $x_s = (\dots, x_{l,s}, \dots)'$; across states of the world, commodities are $(l, s) \in \mathcal{L} \times \mathcal{S}$, and a bundle of commodities is $x = (\dots, x'_s, \dots)' = (\dots, x_{l,s}, \dots)'$.

Assets for the transfer of revenue across states of the world are $a \in \mathcal{A} = \{1, \dots, A\}$, a finite set, and a portfolio of assets is $y = (\dots, y_a, \dots)'$. All assets are numeraire: their payoffs are denominated in units of commodity 1. The payoff of asset a , in units of commodity 1, at the state of the world s is $r_{a,s}$; across states of the world, the payoffs of an asset are $r_a = (\dots, r_{a,s}, \dots)'$. The payoffs of assets at the state of the world s are $R_s = (\dots, r_{a,s}, \dots)$; across states of the world, the matrix of payoffs of assets is

$$R = (\dots, r_a, \dots) = (\dots, R_s, \dots)'.$$

The payoff of a portfolio of assets, y , at state of the world s is $r_s y$; across states of the world, the payoffs of a portfolio of assets are

$$Ry = (\dots, r_s y, \dots).$$

The column span of the matrix of payoffs of assets, $[R]$, is the subspace of attainable reallocations of revenue across states of the world.

Assumption 1. The asset structure is such that

1. there are at least two assets: $A \geq 2$;
2. the matrix of payoffs of assets has full column rank;
3. the payoff of asset $a = 1$ is positive: $r_1 > 0$.
4. at every state of the world, the payoffs of assets do not vanish: $R_s \neq 0$.

Redundant assets, whose payoffs are linear combination of the payoffs of other assets, can be priced by arbitrage.

The asset market is either complete: $A = S$, or incomplete: $A < S$.

A portfolio of assets with positive payoffs serves to eliminate satiation over portfolios; that this portfolio consist of only one asset, $a = 1$, simplifies the exposition.

The preferences of an individual are described by the utility function, w^i , with domain the consumption set.

The preferences of the individual admit a representation that is additively separable across states of the world, $\{u_s^i : s \in \mathcal{S}\}$: the consumption set has a product structure; the cardinal utility function, at a state of the world, is u_s^i , and the utility function is $w^i = \sum_{s \in \mathcal{S}} u_s^i$. The preferences may, but need not admit a von Neumann–Morgenstern representation, (u^i, μ^i) : the consumption sets at different states of the world and the cardinal utility functions, u^i , coincide; $\mu^i = (\dots, \mu_s^i, \dots)$, is a subjective probability measure on the set of states of the world, and the utility function is $w^i = E_{\mu} u^i$.²

Utility functions, w_1^i and w_2^i , are cardinally equivalent if w_2^i is a monotonically increasing, affine transformation of w_1^i .

The endowment of an individual in commodities is e^i , a bundle of commodities across states of the world; his endowment of assets is f^i , a portfolio of assets.

The effective endowment of the individual in commodities is $\tilde{e}^i = (\dots, \tilde{e}_s^i, \dots)'$, where $\tilde{e}_s^i = e_s^i + \mathbf{1}_1^L R_s f^i$ is the effective endowment in commodities at a state of the world.³

²“ E_{μ} ” denotes the expectation with respect to the probability measure μ .

³“ $\mathbf{1}_k^K$ ” denotes the k th unit vector of dimension K .

Assumption 2. For every individual and for every state of the world,

1. the consumption set is the set of nonnegative bundles of commodities;
2. the cardinal utility function, u_s^i , is continuous and concave; in the interior of the consumption set, the utility function is differentially strictly monotonically increasing, $Dv_s^i(x) \gg 0$, and strictly concave, $y \neq 0 \Rightarrow y'D^2u_s^i(x) y < 0$; for a sequence of strictly positive consumption bundles, $(x_{s,n} \gg 0; n = 1, \dots)$, and for \bar{x}_s , a consumption bundle on the boundary of the consumption set, $(\lim_{n \rightarrow \infty} x_{s,n} = \bar{x}_s) \rightarrow (\lim_{n \rightarrow \infty} (\|Du^i(x_{s,n})\|)^{-1} Du^i(x_{s,n}) x_{s,n}) = 0$, while $\lim_{n \rightarrow \infty} \|Du^i(x_{s,n})\| = \infty$;
3. $\tilde{z}^i \gg 0$: the effective endowment in commodities is a consumption bundle in the interior of the consumption set.

The distinction between endowments in commodities and endowments in assets simplifies the exposition, but is not essential.

The profile of utilities functions is

$$u^{\mathcal{J}} = (\dots, \{u_s^i : s \in \mathcal{J}\}, \dots),$$

and the allocation of endowments is

$$(e^{\mathcal{J}}, f^{\mathcal{J}}) = (\dots, (e^i, f^i), \dots).$$

Profiles of utility functions, $u_1^{\mathcal{J}}$ and $u_2^{\mathcal{J}}$, are cardinally equivalent if, for every individual, the utility functions w_1^i and w_2^i are cardinally equivalent.

The profile of utility functions, $u^{\mathcal{J}}$, and the matrix of payoffs of assets, R , are fixed, while the allocation of endowments varies.

The aggregate endowment in commodities is $e^a = \sum_{i \in \mathcal{J}} e^i$, and the aggregate endowment in assets is $f^a = \sum_{i \in \mathcal{J}} f^i$; the aggregate endowment is (e^a, f^a) .

At a state of the world, s , prices of commodities are $p_s = (1, p_{2,s}, \dots, p_{l,s}, \dots) \gg 0$: commodity $l = 1$ is numeraire. Across states of the world, prices of commodities are $p = (\dots, p_s, \dots)$.

Across states of the world, the expenditures associated with bundle of commodities, x , at prices of commodities p are

$$p \otimes x = (\dots, p_s x_s, \dots).$$

Prices of assets are $q = (1, q_2, \dots, q_a, \dots)$: asset $a = 1$ is numeraire.

Prices of assets do not allow for arbitrage if $Ry > 0 \rightarrow qy > 0$; this is the case if and only if $q = \pi R$, for some $\pi = (\dots, \pi_s, \dots) \gg 0$.

Prices are a pair, (p, q) , of prices of commodities and of prices of assets.

The optimization problem of an individual is

$$\max w^i(x), \quad \text{s.t.} \quad p \otimes x \leq p \otimes e^i + Ry, \quad qy \leq qf^i.$$

The solution of the individual optimization problem, $(x^i, y^i)(p, q, e^i, f^i)$, exists and is unique, and the consumption plan, $x^i(p, q, e^i, f^i)$, lies in the interior of the consumption set; it defines (x^i, y^i) , the demand function of the individual for consumption plans and portfolios of assets.

The demand function is continuously differentiable; price effects are

$$\begin{aligned} D_{p_t} x_s^i &= \left(\dots, \frac{\partial x_{l,s}^i}{\partial p_{k,t}}, \dots \right), & D_q x_s^i &= \left(\dots, \frac{\partial x_{l,s}^i}{\partial q_a}, \dots \right), \\ D_{p_{l,s}} y^i &= \left(\dots, \frac{\partial y_a^i}{\partial p_{l,s}}, \dots \right), & D_q y^i &= \left(\dots, \frac{\partial y_a^i}{\partial q_b}, \dots \right); \end{aligned}$$

income effects are

$$\begin{aligned} D_{e_{1,t}^i} x_s^i &= \left(\dots, \frac{\partial x_{l,s}^i}{\partial e_{1,t}^i}, \dots \right), & D_{f_1^i} x_s^i &= \left(\dots, \frac{\partial x_{l,s}^i}{\partial f_1^i}, \dots \right), \\ D_{e_{1,s}^i} y^i &= \left(\dots, \frac{\partial y_a^i}{\partial e_{1,s}^i}, \dots \right), & D_{f_1^i} y^i &= \left(\dots, \frac{\partial y_a^i}{\partial f_1^i}, \dots \right). \end{aligned}$$

For cardinally equivalent utility functions, the demand functions coincide.

Associated with the individual optimization problem, at each state of the world, there is a conditional optimization problem

$$\max u_s^i(z_s), \quad \text{s.t.} \quad p_s z_s \leq p_s e_s^i + R_s y^i,$$

where $y^i > 0$ is a fixed portfolio of assets, such that $p_s e_s^i + R_s y^i > 0$.

The solution of the auxiliary optimization problem, $z_s^i(p_s, e^i, y^i)$, exists, is unique, and lies in the interior of the consumption set; it defines z_s^i , the conditional demand function of the individual.

The conditional demand functions are continuously differentiable; price effects are

$$D_{p_s} z_s^i = \left(\dots, \frac{\partial z_{l,s}^i}{\partial p_{k,s}}, \dots \right);$$

income effects are

$$D_{e_{1,s}^i} z_s^i = \left(\dots, \frac{\partial z_{l,s}^i}{\partial e_{1,s}^i}, \dots \right),$$

and

$$D_{y^i} z_s^i = D_{e_{1,s}^i} z_s^i R_s.$$

The necessary and sufficient conditions for a solution of the conditional individual optimization problem at a state of the world are

$$\begin{aligned} Du_s^i - \lambda_s^i p_s &= 0, \\ p_s z_s^i - p_s e_s^i - R_s y^i &= 0. \end{aligned}$$

Differentiating the first-order conditions and setting

$$\begin{pmatrix} K_s^i & -v_s^i \\ -v_s^{i'} & b_s^i \end{pmatrix} = \begin{pmatrix} D^2 u_s^i & -p_s' \\ -p_s & 0 \end{pmatrix}^{-1},$$

and

$$S_s^i = \lambda_s^i K_s^i,$$

yields, by the implicit function theorem, that

$$\begin{aligned} D_{p_s} z_s^i &= S_s^i - v_s^i (z_s^i - e_s^i)', \\ D_{e_{1,s}^i} z_s^i &= v_s^i, \\ D_{y^i} z_s^i &= v_s^i R_s, \end{aligned}$$

and, as a consequence

$$dz_s^i = (S_s^i - v_s^i (z_s^i - e_s^i)') dp + v_s^i (p_s de_s^i + R_s dy^i).$$

The matrix, S_s^i , of substitution effects, $s_{l,k,s}^i = (\partial x_{l,s}^i / \partial p_{k,s})_{\bar{v}_s^i}$, is symmetric and negative semidefinite; it has rank $(L-1)$ and satisfies $p_s S_s^i = 0$, and the vector, v_s^i , of income effects, $v_{l,s}^i = \partial x_{l,s}^i / \partial t_s^i$, satisfies $p_s v_s^i = 1$.

The necessary and sufficient first-order conditions for a solution to the individual portfolio choice problem are

$$\begin{aligned} \lambda^i R &= \mu^i q, \\ qy - qf^i &= 0, \end{aligned}$$

where, across states of the world, $\lambda^i = (\dots, \lambda_s^i, \dots)$ are the marginal utilities of revenue obtained from the conditional optimization with $y^i = y$.

Differentiating the first-order conditions and setting

$$\begin{pmatrix} K^i & -v^i \\ -v^{i'} & b^i \end{pmatrix} = \begin{pmatrix} -\sum_{s \in \mathcal{S}} b_s^i R_s R_s' & -q \\ -q' & 0 \end{pmatrix}^{-1},$$

and

$$S^i = \lambda^i K^i,$$

yields by the implicit function theorem that

$$D_{p_s} y^i = S^i R_s (\lambda_s^i v_s^{i'} - b_s^i z_s^{i'}),$$

$$D_q y^i = S^i - v^i (y^i - f^i)'$$

$$D_{f_1^i} y^i = v^i,$$

where, for each state of the world, v_s^i and b_s^i are the income effects and the derivative of the marginal utility of revenue, respectively, obtained from the conditional optimization with $y^i = y$.

3. IDENTIFICATION

One considers sequentially identification from individual demand, aggregate demand and equilibrium prices.

3.1. Individual Demand

The individual demand for assets identifies the utility function if there are multiple commodities at each state of the world and prices in the spot markets for commodities enter the demand function for assets. This is the case even if the endowment of the individual in commodities is held fixed, according to a proposition in Geanakoplos and Polemarchakis [22].

LEMMA 1 (Geanakoplos and Polemarchakis [22]). *If, for an individual, at every state of the world,*

1. *the vectors $z_s^i = (\dots, z_{l,s}^i, \dots)$ and $D_{e_{1,s}^i} z_s^i = (\dots, \partial z_{l,s}^i / \partial e_{1,s}^i, \dots)$ are linearly independent, and*

2. $(D_q y^i + D_{f_1^i} y^i (y^i - f^i)') R_s \neq 0,$

then the demand function for consumption plans and portfolios of assets for a fixed endowment of commodities identifies the utility function of the individual up to cardinal equivalence.

Restrictions on the demand of an individual are assumptions are needed for identification from the individual demand function.

The vector of income effects and the vector of demands are not colinear. This excludes homothetic utility functions. As long as income effects are not constant this condition is satisfied generically. In the demand function for assets, the product of the matrix of substitution effects and the payoffs

of all assets does not vanish; the substitution effect in terms of demand for commodities is nonzero.

3.2. Aggregate Demand

Across individuals,

$$(x^a, y^a)(p, q, e^{\mathcal{J}}, f^{\mathcal{J}}) = \sum_{i \in \mathcal{J}} (x^i, y^i)(p, q, e^i f^i),$$

which defines (x^a, y^a) , the aggregate demand function for consumption plans and portfolios of assets.

For cardinally equivalent profiles of utility functions, the aggregate demand functions coincide.

At each state of the world, for $y^{\mathcal{J}} = (\dots, y^i, \dots)$, a fixed allocation of portfolios of assets, such that $p_s e_s^i + R_s y^i > 0$, for every individual,

$$z_s^a(p_s, e_s^{\mathcal{J}}, y^{\mathcal{J}}) = \sum_{i \in \mathcal{J}} z_s^i(p_s, e_s^i, y^i),$$

which defines z_s^a , the aggregate, conditional demand function.

Assumption 3. For every individual,

1. the income effect for every asset, $\partial y_a^i / \partial f_1^i$, is a twice differentiable function of revenue, f_1^i ; Furthermore

$$\frac{\partial^2 y_a^i}{\partial (f_1^i)^2} \neq 0,$$

2. there exist assets, d and e , other than the numeraire, such that

$$\frac{\partial}{\partial f_1^i} \left(\ln \frac{\partial^2 y_d^i}{(\partial f_1^i)^2} \right) \neq \frac{\partial}{\partial f_1^i} \left(\ln \frac{\partial^2 y_e^i}{(\partial f_1^i)^2} \right);$$

for every state of the world,

3. the income effect in the conditional demand for every commodity, $\partial z_{l,s}^i / \partial e_{1,s}^i$, is a twice differentiable function of revenue, $e_{1,s}^i$; and

$$\frac{\partial^2 z_{m,s}^i}{\partial (e_{1,s}^i)^2} \neq 0,$$

4. there exist commodities, m and n , other than the numeraire, such that

$$\frac{\partial}{\partial e_{1,s}^i} \left(\ln \frac{\partial^2 z_{m,s}^i}{(\partial e_{1,s}^i)^2} \right) \neq \frac{\partial}{\partial e_{1,s}^i} \left(\ln \frac{\partial^2 z_{n,s}^i}{(\partial e_{1,s}^i)^2} \right).$$

This is the analogue of the condition of nonvanishing income effects that was employed in the argument under certainty in Chiappori [10]. However, while this assumption translates naturally into an assumption on the rank of the demand system, it is not standard for the demand for assets. Ruling out homothetic utility is necessary but not sufficient. In particular, for the case of only one commodity per state, it rules out constant absolute risk aversion as well as quadratic utility. (We will show below that for this case identification of the utility function is in general not possible. However, under Assumption 3, the identification of individual demand is possible.)

PROPOSITION 1. *For a fixed profile of endowments in commodities, the aggregate demand function for consumption plans and portfolios of assets identifies the profile of utilities up to cardinal equivalence.*

Proof. By Lemma 1, it suffices that the aggregate demand function identifies, for every individual, the demand functions for portfolios of assets, y^i , and z_s^i , the conditional demand function for commodities, at every state of the world.

The argument is developed in two steps.

Step 1. The aggregate demand for assets identified individual asset demands.

For the aggregate demand function for portfolios of assets,

$$D_q y^a = \sum_{i \in \mathcal{I}} (S^i - v^i (y^i - f^i)'),$$

$$D_{f_1^i} y^a = v^i,$$

and, as a consequence

$$dy^a = \sum_{i \in \mathcal{I}} (S^i - v^i (y^i - f^i)') dp + \sum_{i \in \mathcal{I}} v^i df_1^i.$$

Since $D_{f_1^i} y^a = v^i$, the aggregate demand function for portfolios of assets identifies v^i , the income effects of every individual.

The function

$$f_{b,c} = \frac{\partial y_b^a}{\partial q_c} - \frac{\partial y_c^a}{\partial q_b} - \sum_{i \in \mathcal{I}} (v_b^i f_c^i - v_c^i f_b^i), \quad b, c \in \mathcal{A} \setminus \{1\}, \quad b \neq c,$$

for pairs of distinct assets other than the numeraire, is identified by the aggregate demand function.

By direct substitution and the symmetry of the matrices of substitution effects,

$$f_{b,c} = \sum_{i \in \mathcal{I}} (v_c^i f_b^i - v_b^i f_c^i).$$

As in the proof of Lemma 2 from Chiappori [10], the first and second derivatives of the functions $f_{b,c}$ with respect to revenue, f_1^i , identify y^i , the demand function of the individual for portfolios of assets.

Step 2. The aggregate demand function for consumption plans and portfolios of assets, (x^a, y^a) , determines the aggregate, conditional demand function, z_s^a , at every state of the world.

This follows from the fact that the individual demand function determines conditional individual demand: By the supporting hyperplane theorem, given prices of commodities and a portfolio of assets revenue, (p_s, y^i) , there exist commodity prices, p_t , for $t \in \mathcal{S} \setminus \{s\}$, states of the world other than s , and prices of assets, q , such that $y^i(p, q) = y^i$. It follows that $x_s^i(p, q) = z_s(p_s, e_s^i, y^i)$.

For the aggregate, conditional demand function,

$$D_{p_s} z_s^a = \sum_{s \in \mathcal{S}} (S_s^i - v_s^i (z_s^i - e_s^i)'),$$

$$D_{y^i} z_s^a = v_s^i R_s,$$

and, as a consequence

$$dz_s^a = \sum_{s \in \mathcal{S}} (S_s^i - v_s^i (z_s^i - e_s^i)') dp + \sum_{s \in \mathcal{S}} v_s^i (p_s de_s^i + R_s dy^i).$$

Since $D_{y^i} z_s^i = v_s^i R_s$, and since $R_s \neq 0$, the aggregate, conditional demand function identifies, v_s^i , the income effects of every individual.

The functions

$$f_{j,k,s} = \frac{\partial z_{j,s}^a}{\partial p_{k,s}} - \frac{\partial z_{k,s}^a}{\partial p_{j,s}} - \sum_{i \in \mathcal{I}} (v_{j,s}^i e_{k,s}^i - v_{k,s}^i e_{j,s}^i), \quad j, k \in \mathcal{L} \setminus \{1\}, \quad j \neq k,$$

for pairs of distinct commodities other than the numeraire, are identified by the aggregate demand function.

By direct substitution and the symmetry of the matrices of substitution effects,

$$f_{j,k,s} = \sum_{i \in \mathcal{I}} (v_{k,s}^i e_{j,s}^i - v_{j,s}^i e_{k,s}^i).$$

As in the proof of Lemma 2 in Chiappori *et al.* [10] Assumptions 4.3 and 4.4 ensure that the first and second derivatives of the functions $f_{j,k,s}$ with respect to revenue, $e_{1,s}^i$, or, alternatively, y_i , identify z_s^i , the conditional demand function of the individual. ■

Remark. Given the conditional aggregate demand functions for every state, it follows directly from Chiappori *et al.* [10] that all u_s^i can be identified up to a monotone transformation. However, this obviously does not identify the w^i —the aggregate demand function for assets has to be taken into account.

3.3. Identification and Equilibrium

Competitive equilibrium prices, (p, q) , are such that

$$(x^a(p, q, e^{\mathcal{S}}, f^{\mathcal{S}}), y^a(p, q, e^{\mathcal{S}}, f^{\mathcal{S}})) - (e^a, f^a) = 0.$$

The competitive equilibrium correspondence associates competitive equilibrium prices of commodities and assets to profiles of endowments,

$$\omega(e^{\mathcal{S}}, f^{\mathcal{S}}) = \left\{ (p, q) : \begin{array}{l} (x^a(p, q, e^{\mathcal{S}}, f^{\mathcal{S}}), y^a(p, q, e^{\mathcal{S}}, f^{\mathcal{S}})) - (e^a, f^a) = 0, \\ p_{1,s} = 1, s \in \mathcal{S}, q_1 = 1 \end{array} \right\}.$$

For cardinally equivalent profiles of utility functions, the competitive equilibrium correspondences coincide.

PROPOSITION 2. *The competitive equilibrium correspondence on an open set of endowments identifies the associated subset of the consumption sets of individual and the profile of utility functions, up to ordinal equivalence, on this set.*

Proof. It suffices that the competitive equilibrium correspondence identify the profile of demand functions.

The argument is developed in two steps.

Step 1. For an allocation of portfolios of assets, $y^{\mathcal{S}} = (\dots, y^i, \dots)$, the aggregate portfolio of assets is $y^a = \sum_{i \in \mathcal{S}} y^i$.

The graph of the competitive equilibrium correspondence determines the graph of the conditional competitive equilibrium correspondence at every state of the world, which assigns competitive equilibrium prices of commodities to allocations of endowments and portfolios of assets,

$$\omega_s(e_s^{\mathcal{S}}, y^{\mathcal{S}}) = \{ p_s : z_s^a(p_s, e_s^{\mathcal{S}}, y^{\mathcal{S}}) - e_s^a - \mathbf{1}_1^L R_s y^a = 0, \text{ and } p_{1,s} = 1 \}.$$

The graph of the conditional competitive equilibrium correspondence has the structure of a continuously differentiable manifold.

The tangent space to the conditional competitive equilibrium manifold is defined by

$$dz_s^a = de_s^a + \mathbf{1}_1^L R_s dy^a,$$

and, as a consequence, by

$$\sum_{i \in \mathcal{I}} (S_s^i - v_s^i (z_s^i - e_s^i)') dp_s + \sum_{i \in \mathcal{I}} (v_s^i p - I) de_s^i + \sum_{i \in \mathcal{I}} (v_s^i - \mathbf{1}_1^L) R_s dy^i = 0.$$

The conditional competitive equilibrium correspondence determines the competitive equilibrium manifold and, consequently, everywhere, its tangent space.

As in the proof of Proposition 1, at every state of the world, the graph of the conditional competitive equilibrium correspondence identifies the conditional demand function of every individual.

Step 2. By substitution, the tangent space to the competitive equilibrium manifold satisfies

$$\begin{aligned} & \sum_{s \in \mathcal{S}} \left(\sum_{i \in \mathcal{I}} S_s^i R_s (\lambda_s^i v_s^i - b_s^i z_s^{i'}) \right) dp_s \\ & + \sum_{i \in \mathcal{I}} (S^i - v^i (y^i - f^i)') dq + \sum_{i \in \mathcal{I}} (v^i q - I) df^i = 0 \end{aligned}$$

As in the proof of Proposition 1, the graph of the competitive equilibrium correspondence identifies the demand function for assets of every individual and therefore, with Step 1, the entire demand function. ■

Remark. If for every individual, for every commodity and for every state of the world, for any sequence $((p_{s,n}, e_{s,n}^i, y_s^i) : n = 1, \dots)$, of prices of commodities, endowments of commodities and portfolios of assets,

$$\lim_{n \rightarrow \infty} e_{1,s,n}^i + R_s y_s^i = \infty \Rightarrow \lim_{n \rightarrow \infty} z_{l,s,n}^i(p_{s,n}, e_{s,n}^i, y_s^i) = \infty, \quad l \in \mathcal{L};$$

i.e., for every individual, every commodity is normal in a strong sense then the competitive equilibrium correspondence identifies the utility functions of individuals on their entire domain of definition.

Remark. For identification, one need not observe how commodity spot prices at state s vary as endowments in state $t \neq s$ vary. This follows directly from the separability of the utility function.

It is an open question whether identification under uncertainty and an incomplete asset market extends to nonseparable preferences when there are several commodities.

3.4. *The Special Case of a Finance Economy*

The identification result requires that there are at least three commodities at each state of the world.

As is the case under certainty the case of two commodities remains an open question.

The case of one commodity, vacuous under certainty, is indeed of interest under uncertainty.

Although recoverability from individual demand requires additional assumptions (see Dybvig and Polemarchakis [18]), the recoverability of individual allocations from the competitive equilibrium correspondence is not problematic.

Step 2 of the proof of Proposition 2 implies that—as long as parts (1) and (2) of Assumption 4 are satisfied—individual asset demand can be identified from the equilibrium manifold even if $L = 1$.

However, in this framework individual asset demand as a function of individual endowments at all states and of prices does not identify the utility.

If $\mathbf{L} = 1$, the individual asset demand function is a solution to

$$\max w^i(Ry), \quad \text{s.t.} \quad qy \leq qf^i.$$

Differentiating the first-order conditions and setting

$$\begin{pmatrix} K^i & -v^i \\ -v^{i'} & b^i \end{pmatrix} = \begin{pmatrix} -R'_s D^2 w^i R_s & -q \\ -q' & 0 \end{pmatrix}^{-1},$$

one obtains that $D^2 w^i$ can be recovered by variation of e_i^s after the identification of K^i and v^i from variation in prices and endowments in portfolios, since $D^2 w^i$ is a diagonal matrix.

However, the Hessian of the utility function generally does not identify preferences.

In particular, whenever $J < S$ there exists a S -dimensional vector h , perpendicular to R , such that the utility functions $w(x)$ and $\tilde{w}(x) = hx + w(x)$ generate the same demand for assets.

The example shows it is possible that, even though equilibrium allocations are inefficient, a planner who can observe the equilibrium correspondence but not individual preferences might not be able to introduce Pareto-improving assets; a firm might not be able to choose a constrained efficient production plan.

Nevertheless, if it is assumed that $w(x)$ has a von Neumann–Morgenstern representation, that there is a riskless asset, and that the correspondence allows for globally recovering the demands, the arguments in Dybvig and Polemarchakis [18] simply that preferences can be recovered as well.

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