A Role for Monetary Policy when Prices Reveal Information: An Example[1]

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In an example, monetary policy can determine the information revealed by prices, and thus it can be effective. Monetary policy that varies only with public information can guarantee the full revelation of information at equilibrium, which is optimal. Full indexation need not attain full revelation and, as a consequence, need not implement an optimal allocation. *Journal of Economic Literature* Classification Numbers: D52, D82, E52.   © 2000 Academic Press

1. INTRODUCTION

Money serves as a unit of account or numeraire; it also serves as a medium of exchange and a store of value, though this is secondary. Substantive arguments for effective monetary policy have focused on money as a unit of account.

Monetary policy can have real effects by altering the structure of payoffs of assets, which determines the attainable reallocations of revenue across contingencies: it alters the level of prices which, in turn, determines the real payoff of nominal assets; the allocations of resources at equilibrium associated with distinct asset structures are typically distinct. This argument, which had been put forward by Tobin [17], was formalized in the

literature on equilibria with an incomplete asset market and nominal assets, first by Cass [2] and then by Balasko and Cass [1], Geanakoplos and Mas-Colell [8], and the literature that followed. That the asset market be incomplete is necessary: with a complete asset market, variations in the structure of payoffs of assets can only be inessential. It is also necessary that some assets be nominal: it was shown by Chamley and Polemarchakis [3] that, with indexed assets, variations in the supply of money and the level of prices fail to alter essentially the structure of payoffs of assets and are neutral, which parallels the argument of Modigliani and Miller [13], in corporate finance. The specification of the standard model of an economy with an incomplete asset market in which money balances are in zero net supply and monetary policy reduces to the exogenous determination of the level of prices is an abstraction: the features and conclusions of the model carry over in tact either in economies, following Clower [5], with a cash-in-advance technology of transactions, as in Dubey and Geanakoplos [7] and Magill and Quinzii [11], or, following Samuelson (1958), in economies with a demographic structure of overlapping generations, which allow for money balances in positive net supply, as in Detemple *et al.* [6].

When individuals are asymmetrically informed, rationality in the formation of expectations, following Radner [15], requires that individuals refine their private information with the information revealed by prices. Prices of assets and commodities do not convey only the aggregate scarcity of resources: they convey information across individuals.

If individuals form expectations rationally, monetary policy is neutral unless changes in the supply of money are stochastic and prevent individuals from distinguishing real from monetary shocks; this was the argument of Lucas [10]. Alternatively, as shown by Weiss [18], monetary policy can be effective by allowing prices to reveal information private to some individuals.

The information revealed by prices depends on the structure of assets available for the transfer of revenue across date-events. As shown by Chamley [4], different asset structures lead to qualitatively distinct behavior of macroeconomic variables.

Earlier work by Mischel *et al.* [12], Polemarchakis and Siconolfi [14], and Rahi [16] exploited the indeterminacy of equilibrium when assets are nominal and the asset market is incomplete to show that, when individuals are differentially informed, non-informative rational expectations equilibria exist.

Here, an example illustrates that monetary policy can determine the information revealed by prices, and, thus, it can have real effects.

There exists an open set of distinct, competitive equilibrium allocations associated with fully non-informative rates of interest and a unique

competitive equilibrium allocation associated with fully revealing rates of interest.

By construction, the equilibrium allocation with full revelation of information coincides with the allocation that would have been obtained had individuals had access to a complete system of insurance contracts prior to the acquisition of private information: the effects pointed out by Hirschleifer [9] are absent.

Monetary policy that varies only with public information can guarantee the full revelation of information at equilibrium, which is optimal.

A generic set of monetary policies indeed attains full revelation but this is not compelling, since monetary policy is not an exogenous, structural parameter.

Full indexation need not attain full revelation and, as a consequence, need not implement an optimal allocation.

2. EFFECTIVE MONETARY POLICY

The example is minimal.

Dates are $t = 1, 2$.

A single, perishable, non-produced commodity is exchanged and consumed at each date; quantities of the commodity are denoted x_t.

Individuals are $i = 1, 2$. They derive utility according to the intertemporal utility functions

$$x_1 + E(x_2 - \tfrac{1}{2}\sigma^i x_2^2).$$

They are endowed with $e^i > 0$ units of the commodity at date 2—the endowment at date 1 is immaterial.

The coefficients of risk aversion,

$$\sigma^1 = \alpha > 0 \qquad \text{and} \qquad \sigma^2 = \beta > 0,$$

of individuals are random; monotonicity of the utility functions of individuals requires that $\alpha < 1/(e^1 + e^2)$ and $\beta < 1/(e^1 + e^2)$. There are finitely many states of the world, realizations of the coefficients of risk aversion,

$$(\alpha, \beta) \in \{(\alpha_k, \beta_n) : k = 1, ..., K, n = 1, ..., N\}.$$

The probability of a state is $\pi(\alpha, \beta)$, while the conditional probabilities are $\pi(\alpha \mid \beta) > 0$ and $\pi(\beta \mid \alpha) > 0$.

Economies are indexed by the endowments of individuals and the values of the coefficients of risk aversion at date 2.

Individuals trade in a nominally riskless bond in order to transfer wealth across dates and, indirectly, across contingencies.

At date 1, individuals exchange the commodity against the bond that pays off at date 2; holdings of the bond are denoted y. The nominal rate of interest is r.

Monetary policy determines the price level at date 2; equivalently, it determines the price or purchasing power of units of account, p.

At date 1, when trade occurs, individuals are asymmetrically informed: each receives a signal, σ^i, the realization of his or her coefficient of risk aversion, but receives no information concerning the coefficient of risk aversion of the other individual. At date 2, the state of the world is public knowledge.

Individuals refine their information at date 1 by the information revealed by the rate of interest.

Fully Revealing Equilibria

Rates of interest across states of the world are fully revealing if

$$(\alpha, \beta) \neq (\alpha', \beta') \Rightarrow r(\alpha, \beta) \neq r(\alpha', \beta').$$

If rates of interest are fully revealing, the optimization problem of individual 1 at (α, β) is

$$\max_{y} -\frac{y}{1 + r(\alpha, \beta)} + p(\alpha, \beta)\, y + e^1 - \frac{1}{2}\alpha(p(\alpha, \beta)\, y + e^1)^2,$$

and his or her portfolio demand is

$$y^1(\alpha, \beta) = (1 - \alpha e^1)(\alpha p(\alpha, \beta))^{-1} - \left(\frac{1}{1 + r(\alpha, \beta)}\right)(\alpha p^2(\alpha, \beta))^{-1};$$

similarly, the portfolio demand of individual 2 is

$$y^2(\alpha, \beta) = (1 - \beta e^2)(\beta p(\alpha, \beta))^{-1} - \left(\frac{1}{1 + r(\alpha, \beta)}\right)(\beta p^2(\alpha, \beta))^{-1}.$$

Equilibrium rates of interest are such that

$$y^1(\alpha, \beta) + y^2(\alpha, \beta) = 0$$

or

$$\frac{1}{1 + r(\alpha, \beta)} = p(\alpha, \beta)\,\frac{(\alpha + \beta - \alpha\beta(e^1 + e^2))}{(\alpha + \beta)}.$$

CLAIM 1. *There is a unique equilibrium allocation of commodities associated with fully revealing rates of interest.*

Proof. By direct substitution, the consumption, at (α, β), by individual 1 at date 2 is

$$x_2^1(\alpha, \beta) = \frac{\beta(e^1 + e^2)}{\alpha + \beta},$$

and by individual 2 it is

$$x_2^2(\alpha, \beta) = \frac{\alpha(e^1 + e^2)}{\alpha + \beta}. \quad \blacksquare$$

The purchasing power of units of account is immaterial beyond garanteeing full revelation.

For the price of the asset to be fully revealing it is necessary and sufficient that the purchasing power of the units of account satisfy

$$\frac{p(\alpha, \beta)}{p(\alpha', \beta')} \neq \frac{(\alpha' + \beta' - \alpha'\beta'(e^1 + e^2))}{(\alpha + \beta - \alpha\beta(e^1 + e^2))} \frac{(\alpha + \beta)}{(\alpha' + \beta')}, \qquad (\alpha, \beta) \neq (\alpha', \beta').$$

Full indexation stabilizes the price level:

$$p(\alpha, \beta) = p.$$

If there exist distinct states of the world,

$$(\alpha, \beta) \neq (\alpha', \beta'),$$

such that

$$\frac{(\alpha + \beta - \alpha\beta(e^1 + e^2))}{(\alpha + \beta)} = \frac{(\alpha' + \beta' - \alpha'\beta'(e^1 + e^2))}{(\alpha' + \beta')},$$

full indexation is not compatible with full revelation.

Fully Non-informative Equilibria

Rates of interest across states of the world are fully non-informative if

$$r(\alpha, \beta) = r.$$

If the rate of interest is fully non-informative, the optimization problem of individual 1 at (α, β) is

$$\max_{y} -\frac{y}{1+r} + E_{\beta \mid \alpha}\left[\, p(\alpha, \beta)\, y + e^1 - \frac{1}{2}\, \alpha(p(\alpha, \beta)\, y + e^1)^2\right],$$

and his or her portfolio demand is

$$y(\alpha) = \left[\, E_{\beta \mid \alpha}p(\alpha, \beta) - \alpha e^1 E_{\beta \mid \alpha}p(\alpha, \beta) - \frac{1}{1+r}\right]\,[\alpha E_{\beta \mid \alpha}p(\alpha, \beta)^2]^{-1};$$

similarly, the portfolio demand of individual 2 is

$$y(\beta) = \left[\, E_{\alpha \mid \beta}p(\alpha, \beta) - \beta e^2 E_{\alpha \mid \beta}p(\alpha, \beta) - \frac{1}{1+r}\right]\,[\beta E_{\alpha \mid \beta}p(\alpha, \beta)^2]^{-1}.$$

The equilibrium rate of interest is such that

$$y(\alpha) + y(\beta) = 0.$$

CLAIM 2. *For an open set of economies, the set of distinct equilibrium allocations associated with fully non-informative rates of interest contains an open set of dimension* $(K-1)(N-1)$.

Proof. The system of equations

$$H = (f_k : k = 1, ..., K,\ g_n : n = 1, ..., N) = 0,$$

in the variables

$$(\alpha, \beta, p, r, y) = (\alpha_k, \beta_n, p(\alpha_k, \beta_n)) : k = 1, ..., K, n = 1, ..., N, r, y),$$

is defined by

$$f_k = \alpha E_{\beta \mid \alpha}p(\alpha, \beta)^2\, y - E_{\beta \mid \alpha}p(\alpha, \beta) + \alpha e^1 E_{\beta \mid \alpha}p(\alpha, \beta) + \frac{1}{1+r},$$

$$g_n = \beta E_{\alpha \mid \beta}p(\alpha, \beta)^2(-y) - E_{\alpha \mid \beta}p(\alpha, \beta) + \beta e^2 E_{\alpha \mid \beta}p(\alpha, \beta) + \frac{1}{1+r}.$$

The value $(\bar{\alpha}, \bar{\beta}, \bar{p}, \bar{r}, \bar{y})$ defined by

$$\bar{\alpha}_k = a, \qquad \bar{\beta}_n = b,$$

$$\bar{p}(\alpha_n, \beta_n) = 1,$$

$$\frac{1}{1+\bar{r}} = -\frac{ab}{a+b},$$

$$\bar{y} = \frac{b(e^1 + e^2)}{a+b} - e^1 = e^2 - \frac{a(e^1 + e^2)}{a+b},$$

is a solution; there

$$\frac{\partial f_k}{\partial p(\alpha_k, \beta_N)} = \pi(\beta_N \mid \alpha_k) \left(\frac{2ab(e^1 + e^2)}{a+b} - ae^1 - 1 \right),$$

$$\frac{\partial g_n}{\partial p(\alpha_K, \beta_n)} = \pi(\alpha_K \mid \beta_n) \left(\frac{2ab(e^1 + e^2)}{a+b} - be^2 - 1 \right),$$

$$\frac{\partial f_k}{\partial r} = \frac{\partial g_n}{\partial r} = -\frac{1}{(1+r)^2}.$$

The vector of prices of units of account across states of the world is partitioned according to

$$p = (p_1, p_2),$$

where

$$p_1 = (p(\alpha_k, \beta_n) : k = 1, ..., K-1, n = 1, ..., N-1),$$

and

$$p_2 = (p(\alpha_K, \beta_n) : n = 1, ..., N, p(\alpha_k, \beta_N) : k = 1, ..., K).$$

Since

$$k \neq k' \Rightarrow \frac{\partial f_k}{\partial p(\alpha_{k'}, \beta_n)} = 0,$$

and

$$n \neq n' \Rightarrow \frac{\partial g_n}{\partial p(\alpha_k, \beta_{n'})} = 0,$$

the Jacobian matrix $D_{(p_2, q)} H$ has full rank at $(\bar{\alpha}, \bar{\beta}, \bar{p}, \bar{r}, \bar{y})$, as long as

$$\frac{\partial f_k}{\partial p(\alpha_k, \beta_N)} \neq 0, \qquad \frac{\partial g_n}{\partial p(\alpha_K, \beta_n)} \neq 0$$

and

$$\frac{\partial f_K}{\partial p(\alpha_K, \beta_N)} \neq \frac{\partial g_N}{\partial p(\alpha_K, \beta_N)}$$

or

$$\frac{2ab(e^1 + e^2)}{a + b} - ae^1 - 1 \neq \frac{2ab(e^1 + e^2)}{a + b} - be^2 - 1.$$

By the implicit function theorem, for (α, β, p_1, y) in an open neighborhood of $(\bar{\alpha}, \bar{\beta}, \bar{p}_1, \bar{y})$, there exists (p_2, r), such that $H = 0$. ∎

Without private information, the dimension of the set of distinct equilibrium allocations when the asset market is incomplete and assets are nominal is equal to the cardinality of the set of states of the world minus 1; here, $KN - 1$. In addition $(K-1) + (N-1)$ degrees of freedom are taken up to guarantee that the private information of individuals is not reflected in the first period asset market, and the remaining degrees of freedom, $KN - 1 - (K-1) - (N-1) = KN - K - N + 1 = (K-1)(N-1)$, is the degree of indeterminacy.

An extension of the construction here generates equilibria with partial revelation of information.

3. OPTIMALITY

Once the state of the world has been realized and revealed to all individuals by the prices of the asset, the financial market is complete.

The appropriate vantage point for welfare considerations lies before the acquisition of private information. In the absence of assets that would permit individuals to hedge fluctuations in their degree of risk aversions, the asset market is incomplete.

Nevertheless, at a fully revealing equilibrium, the financial market is effectively complete: in state (α, β), the marginal utility of consumption is constant and equal to 1 at date 1 and equal to

$$1 - \frac{\alpha\beta(e_1 + e_2)}{\alpha + \beta}$$

at date 2 for both individuals; this is due to the constant marginal utility of consumption at date 1.

Monetary policy that guarantees full revelation yields an optimal allocation.

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