

Trade and Welfare ¹

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Abstract

Domestic transfers such that all individuals gain in utility with international trade, compared to domestic autarky, need not exist.

Key words: gains from international trade; domestic transfers.

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The argument for international trade is that in a competitive equilibrium with trade all individuals gain in utility, compared to domestic autarky.

Ever since the modern formulation of gains from trade by Samuelson (1939) and the identification of competitive equilibrium allocations with pareto optimal allocations by Arrow (1951), it has been recognized that the argument requires transfers across individuals. In world economy which consists of many countries, and in which each country is a collection of possibly heterogeneous individuals, domestic autarky refers to a competitive equilibrium allocation of each country in isolation, though not of each individual in isolation. A competitive allocation with trade across countries need not entail an improvement in the utility of every individual. Transfers are thus necessary to guarantee pareto gains from trade, and, under convexity and regularity assumptions, they are also sufficient.

Transfers of commodities or revenue balance across individuals. Domestic transfers balance across individuals within each country. They are thus a restricted class of transfers of evident interest for policy.

Are domestic transfers sufficient for pareto gains from trade?

An affirmative answer was given by Grandmont and McFadden (1972), with an elegant argument. Consider a competitive equilibrium allocation for each country, the autarkic allocation, and a competitive equilibrium allocation for the world economy obtained after the domestic reallocation of commodities associated with the autarkic allocation. By a revealed preference argument, each individual is at least as well off in this competitive equilibrium allocation for the world economy as in autarky. Evaluating the domestic reallocation of commodities at the competitive equilibrium prices for the world economy determines domestic transfers of revenue sufficient for pareto gains from trade.

In a recent paper, Cordella and Ventura (1992) showed that the argument fails if domestic transfers of commodities are implemented after trade in world markets.

Here, we argue that domestic transfers of commodities or revenue implemented before trade in world markets need not suffice for pareto gains from trade.

Our argument complements that argument of Grandmont and McFadden, if by contradicting its conclusion.

In the absence of strong monotonicity and strictly positive prices, which is natural, the existence of competitive equilibria requires, following McKenzie (1959, 1961), the economy be resource related: that any subset of individuals be endowed with commodities which could be employed to increase the utility of the complementary subset at any feasible allocation. Resource relatedness is thus a property of the preferences as well as of the endowments of individuals. Even if the world economy is resource related as is every country in isolation, the world economy may fail to be resource related after the endowments are modified to coincide with an autarkic equilibrium allocation. A competitive equilibrium following the domestic transfer of commodities may then fail to exist, and the argument of Grandmont and McFadden cannot proceed.

When, following the transfer of commodities associated with domestic autarky, resource relatedness fails, and competitive equilibria fail to exist, two

possibilities arise: either an alternative transfer of commodities or revenue exists which leads to pareto gains from trade; or some individual loses in utility with international trade relative to domestic autarky for all redistributions of commodities or revenue compatible with equilibrium.

We construct an example which illustrates both possibilities.

1 The example

A world economy, \mathcal{W} , has individuals $h = 1, 2, 3$, and commodities $l = 1, \dots, 4$. The world economy is divided into countries, \mathcal{A} and \mathcal{B} , each a subset of individuals. In particular, $\mathcal{A} = \{1, 2\}$ and $\mathcal{B} = \{3\}$.

The utility functions and endowments of individuals are as follows:

For country \mathcal{A} ,

$$\begin{aligned} u^1 &= x_1 + x_3 + \min\{1 + \delta, x_4\}, & x, &\geq 0, & \omega^1 &= (1, 1, 1, 1), \\ u^2 &= x_2, & x &\geq 0, & \omega^2 &= (1, 1, 0, 0), \end{aligned}$$

and for country \mathcal{B} ,

$$u^3 = x_1, \quad x \geq 0, \quad \omega^3 = (1, 1, 1, \epsilon),$$

with

$$0 \leq \delta < \epsilon < \frac{1}{2}.$$

The world economy as well as the economy of every country satisfy standard conditions for the existence and optimality of competitive equilibrium allocations: the preferences of individuals described by their consumption sets and utility functions are continuous, convex and display local nonsatiation, and, most importantly, in conjunction with their endowments, they satisfy resource relatedness. An economy satisfies resource relatedness if and only if, for any nontrivial partition of the set of individuals, \mathcal{H} , into two, nonempty subsets, \mathcal{H}_1 and \mathcal{H}_2 , and for any feasible allocation, $x^{\mathcal{H}} = \{x^h : h \in \mathcal{H}\}$, there exists an individual $h \in \mathcal{H}_2$ such that $u^h(x^h + \sum_{h_1 \in \mathcal{H}_1} \omega^{h_1}) > u^h(x^h)$.

Since in both countries, and hence in the world economy as well, at least one individual has a utility function strictly monotonically increasing in the consumption of commodity $\ell = 1$, we treat it through out as numeraire: $p_1 = 1$.

The autarkic equilibrium in country \mathcal{A} is unique and obtains at prices

$$p^{\mathcal{A}} = (1, 1, 1, 1),$$

and the associated allocation of commodities and utilities is

$$\begin{aligned} x^{1,\mathcal{A}} &= (2, 0, 1, 1), & u^{1,\mathcal{A}} &= 4, \\ x^{2,\mathcal{A}} &= (0, 2, 0, 0), & u^{2,\mathcal{A}} &= 2, \end{aligned}$$

while in country \mathcal{B} the unique autarkic equilibrium prices are

$$p^{\mathcal{B}} = (1, 0, 0, 0),$$

and the associated allocation of commodities and utilities is

$$x^{3,\mathcal{B}} = (1, 1, 1, \epsilon), \quad u^{3,\mathcal{B}} = 1.$$

The equilibrium for the world economy \mathcal{W} is unique and obtains at prices

$$p^{\mathcal{W}} = (1, \frac{1}{2}, 1, 0),$$

and the associated allocation of commodities and utilities is

$$\begin{aligned} x^{1,\mathcal{W}} &= (\frac{1}{2}, 0, 2, 1 + \epsilon), & u^{1,\mathcal{W}} &= \frac{7}{2} + \delta, \\ x^{2,\mathcal{W}} &= (0, 3, 0, 0), & u^{2,\mathcal{W}} &= 3, \\ x^{3,\mathcal{W}} &= (\frac{5}{2}, 0, 0, 0), & u^{3,\mathcal{W}} &= \frac{5}{2}. \end{aligned}$$

Individual $h = 1$ is worse off with trade compared with autarky:

$$u^{1,\mathcal{W}} = \frac{7}{2} + \delta < 4 = u^{1,\mathcal{A}}.$$

Feasible allocations for the world economy such that all individuals are strictly better off than at the competitive equilibrium with autarky indeed exist: for example, the allocation

$$\begin{aligned} x^{1,\mathcal{W}} &= (2 - k, 0, 2, 1 + \epsilon), & u^{1,\mathcal{W}} &= 5 = \delta - k, \\ x^{3,\mathcal{W}} &= (0, 3, 0, 0), & u^{2,\mathcal{W}} &= 3, \\ x^{3,\mathcal{W}} &= (1 + k, 0, 0, 0), & u^{3,\mathcal{W}} &= 1 + k. \end{aligned}$$

for

$$0 < k < 1 + \delta.$$

Furthermore, each such allocation can be obtained as a competitive equilibrium with transfers of commodities or, more simply, revenue,

$$\tau^{1,\mathcal{W}} = \frac{3}{2} = k, \tau^{2,\mathcal{W}} = 0, \tau^{3,\mathcal{W}} = k - \frac{3}{2},$$

and prices,

$$p^{\mathcal{W}} = (1, \frac{1}{2}, 1, 0).$$

These, however, are not domestic transfers: they do not balance across individuals within each country.

The question which arises is whether domestic transfers exist for which an associated competitive equilibrium allocation improves the utility of every individual compared to autarky.

Let τ be the transfer of revenue from individual $h = 2$ to individual $h = 1$ in country \mathcal{A} – it is pedantic to write $\tau^1 = \tau$ and $\tau^2 = -\tau$ – while no transfer occurs within country \mathcal{B} which consists of one individual. Since individual $h = 2$ is the only one to demand commodity $\ell = 2$ and he demands only that commodity, his

revenue equals the aggregate expenditure of commodity $\ell = 2$ or $1 + p_2 - \tau = 3p_2$. Thus $2p_2 = 1 - \tau$, which implies that only transfers

$$\tau < 1$$

are possibly compatible with competitive equilibria.

Indeed, following a domestic transfer of revenue $\tau < 1$, in country \mathcal{A} , the equilibrium for the world economy \mathcal{W} is unique and obtains at prices

$$p^{\mathcal{W}} = \left(1, \frac{1 - \tau}{2}, 1, 0\right),$$

and the associated allocation of commodities and utilities is

$$\begin{aligned} x^{1,\mathcal{W}} &= \left(\frac{1+\tau}{2}, 0, 2, 1 + \epsilon\right), & u^{1,\mathcal{W}} &= \frac{7+\tau}{2} + \delta, \\ x^{2,\mathcal{W}} &= (0, 3, 0, 0), & u^{2,\mathcal{W}} &= 3, \\ x^{3,\mathcal{W}} &= \left(\frac{5-\tau}{2}, 0, 0, 0\right), & u^{3,\mathcal{W}} &= \frac{5-\tau}{2}. \end{aligned}$$

For $\delta \in (0, 1/2)$, a transfer $\tau \in (1 - 2\delta, 1)$ yields a competitive equilibrium at which all individuals gain in utility compared to autarky.

For $\delta = 0$, no domestic transfer yields a competitive equilibrium at which all individuals gain in utility compared to autarky.

It is instructive to see that the argument of Grandmont and McFadden fails, and that is the case for $\delta = 0$ as well as for $\delta \in (0, 1/2)$. Consider, as Grandmont and McFadden suggest, the world economy but with initial endowments for individuals in country \mathcal{A}

$$\begin{aligned} \omega^1 &= (2, 0, 1, 1), \\ \omega^2 &= (0, 2, 0, 0). \end{aligned}$$

If $p_2 > 0$, there is an excess supply of commodity $\ell = 2$ equal to 1, the endowment of individual $h = 3$, which is not compatible with market clearing. The only individual who could absorb this excess, individual $h = 2$, has no endowment of another commodity to offer in exchange. If $p_2 = 0$, the optimization problems of individual $h = 2$ has no solution, which, again, is not compatible with market clearing. Grandmont and McFadden explicitly rule out this type of situation by assuming, in the premise of their theorem 3, pg 121, that in each nation, at the autarkic equilibrium allocation, all individuals have positive revenue at the equilibrium world prices. This assumption clearly guarantees that competitive equilibria for the world economy exist when the domestic transfer of commodities equates the initial endowments of individuals with their allocation at the autarkic competitive equilibrium.

This way of proceeding is not totally satisfactory, because the assumption is not stated in terms of the structural characteristics, the fundamentals, of the economy.

Interestingly, even when the transfer suggested by Grandmont and McFadden fails to yield equilibria and a fortiori to a pareto improvement in welfare relative to domestic autarky, other such transfers may exist. In our example, they do for $\delta \in (0, 1/2)$, but not for $\delta = 0$.

2 Conclusion

It is an open question to characterize minimal sufficient conditions for the existence of domestic transfers such that, with international trade, every individual gains in utility compared to domestic autarky.

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