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NOTES, COMMENTS, AND LETTERS TO THE EDITOR

Partial Revelation with Rational Expectations

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We exhibit a partially revealing rational expectations equilibrium, where no individual knows the state of the world or the quantities traded by any other individual. *Journal of Economic Literature* Classification Number: D82.

1. INTRODUCTION

In an economy with asymmetric information, intelligent individuals base their decisions not only on their private information, but also on the prices they observe, together with what they believe to be the set of situations in which they would observe similar prices. When these beliefs are correct and markets clear, the economy is said to be at a rational expectations equilibrium.

Work on rational expectations equilibrium concentrated, initially, following Radner [14], on conditions under which there would remain no uncertainty that individuals could resolve by pooling their information—Jordan and Radner [10] or Allen [2] survey the classical references. Yet, in real-life situations, full revelation during trade is the exception rather than the rule.

With more dimensions of uncertainty than commonly observed prices, Allen [1], Ausubel [3], and Jordan [9] constructed examples of partially revealing rational expectations equilibria. In the finance literature, following Grossman [7], rational expectations equilibria are, for the same reason, typically, partially revealing.

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In order to captures the idea that individuals may not "know" the mapping from states of the world to equilibrium prices, Dutta and Morris [6] allowed prices to follow a nondegenerate distribution conditional on the state of the world and showed that full revelation need not obtain at equilibrium; similarly, Pietra and Siconolfi [11] obtained nonrevelation by expanding the set of states of the world to allow for extrinsic uncertainty; DeMarzo and Skiadas [4] characterized conditions that guarantee full revelation.

In a two period economy with finitely many states of the world and nominal assets, Polemarchakis and Siconolfi [12] also exhibited the existence of a partially revealing rational expectations equilibrium; even stronger, a continuum of fully nonrevealing equilibria.

It is a common features of these constructions, however, that one can partition the domain of uncertainty into equivalence classes of states that are common knowledge at equilibrium. Across states of the world in each of these classes, the prices of goods and the excess demand of each individual are constant. Thus, there remains no asymmetry of information at equilibrium regarding the actions taken by individuals.

Rahi [15] considered another set up of a two-period, nominal asset economy, where, in common knowledge equivalence classes of states with the same first-period prices, different individuals may demand different quantities at equilibrium. However, the construction requires that there be, in each such equivalence class of states of the world, an individual who is fully informed about the joint information of the other individuals. In a world with many heterogeneous sources of uncertainty, such an assumption is not appealing.

In the finance literature, Diamond and Verrecchia [5] presented an economy with a risky asset whose return has a normal distribution, while the endowments of individuals are drawn independently from a normal distribution as well. Each individual gets a noisy signal regarding the payoff of the asset, and, at equilibrium, the same price of the asset holds in states where the return is high while the aggregate endowment is low or vice versa. In particular, at equilibrium, individuals do not get to know the actions of the others. However, the normal distribution of the asset's return implies unbounded liability to its holders.

Here, we construct economies under uncertainty with a partially revealing rational expectations equilibrium without the above shortcomings. In particular, at equilibrium, no individual can tell the true state of the world or the quantities bought or sold by other individuals, and trade is in riskless contingent goods.

In the main example there are three individuals, each of whom observes a real number as a private signal. There are two goods, so individuals further observe one relative price. Typically, at equilibrium, the private signal and the price frame, for each individual, a segment of joint signals of the two other individuals. The utility function of the individual is not constant along the segment, so he is bound to form his demand according to his expected utility there. At equilibrium, the two other individuals demand different quantities along the segment, so the individual cannot tell who is buying, who is selling and how much of each good. There is symmetry in the economy—permuting the signals between individuals would lead to the corresponding permuted behavior. The equilibrium is robust to perturbations in the endowments and in the parameters determining the utility functions, for a compensating perturbation of the domain of uncertainty. The equilibrium price is a smooth function of the joint signals, in the original economy as well as in the perturbed ones. A similar economy with more individuals and goods can be constructed, and the result extends to the case in which the volume of trade is also commonly observed by individuals.

2. THE ECONOMY

There are three individuals, $i=1, 2, 3$. There are two commodities, quantities of which we denote by x and y. The domain of uncertainty or the space of states of the world is

$$T=\{t=(t^1, t^2, t^3): 0 \leqslant t^1+t^2+t^3 \leqslant m\},$$

for some $m>0$.

Endowments of commodities x and y are, respectively, $w_1>0$ and $w_2>0$, the same for each individual and for each state of the world.

The cardinal utility function of an individual,

$$u^i(t)(x, y)=\alpha^i(t) \ln x+[1-\alpha^i(t)] \ln y,$$

depends on the state of the world through the relations

$$\alpha^1(t)=at^1+bt^2+ct^3+d,$$

$$\alpha^2(t)=at^2+bt^3+ct^1+d,$$

$$\alpha^3(t)=at^3+bt^1+ct^2+d,$$

with $0<d<1$, $a, b, c>0$, $b \neq c$, and $\max\{a, b, c\}<(1-d)/m$. The last condition simply guarantees that $0<\alpha^i(t)<1$ throughout the domain of uncertainty.

Individuals have a uniform prior distribution on the domain of states of the world. When trade occurs, individual i observes the coordinate, t^i, and the relative price, $\pi \in (0, \infty)$, of the second good with respect to the first. Knowing the association of prices to states, the individual computes his posterior distribution and expresses demand which maximizes his posterior expected utility subject to his budget constraint. A rational expectations equilibrium is a Borel-measurable map from states to prices, such that the markets for the two goods clear at every state.

We exhibit a rational expectations equilibrium where the set of states in which a given price, π, prevails is a triangle, $T_r = \{t \in T: t^1 + t^2 + t^3 = r\}$, where $0 \leqslant r \leqslant m$. The explicit form of the price function appears in Eq. (1) below. Given a price function of this form, the posterior distribution of individual i, when he gets to know the coordinate $\bar{t}^i$ and the price π, is uniform along the segment

$$S^i(\bar{t}^i, r) = \{t \in T: t^i = \bar{t}^i, t^j + t^k = r - \bar{t}^i\},$$

where j and k are the two other individuals.

For example, the posterior of individual 1 in Fig. 1 is uniformly distributed along the segment AB when he observes $\bar{t}^1$ and π.

Since $\alpha^i(t)$ changes linearly along the segment $S^i(\bar{t}^i, r)$, the posterior expected utility function of individual i is

$$u^i(x, y) = \beta^i \ln x + (1 - \beta^i) \ln y,$$

where β^i is the mean of the values of $\alpha^i(t)$ at the two endpoints of the segment. Thus, one can compute that the demand of the individual is

$$x^i = \beta^i(w_1 + \pi w_2),$$

$$y^i = (1 - \beta^i)\left(\frac{w_1}{\pi} + w_2\right).$$

Along the triangle, β^i is an affine function of $\bar{t}^i$; that is $d\beta^i/d\bar{t}^i$ is constant. Since the price π is the same in the entire triangle, $dx^i/d\bar{t}^i$ and $dy^i/d\bar{t}^i$ are also constant there. Hence, if the markets clear at the corners of the triangle, they also clear at any other point of the triangle: the coordinates of such an inner point are a convex combination of those of the corners, and, thus, the aggregate excess demand there is the same convex combination of the aggregate excess demand at the corners, which is zero.

By our proposed structure for the rational expectations equilibrium, seeing the price, π_0, which prevails at the origin, $(0, 0, 0)$, the individuals know they are there. It is straightforward to compute that this price π_0 is, therefore,

$$\pi_0 = \frac{w_1(1 - d)}{w_2 d}.$$

In our proposed structure for the rational expectations equilibrium, as we move further from the origin along any ray, the utility of individuals

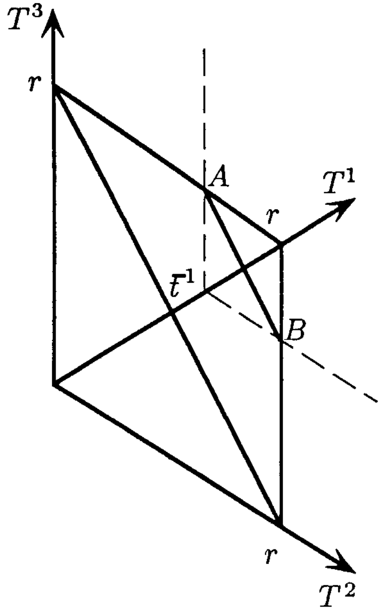

Figure 1

increases as a function of the first good and decreases as a function of the second. (See Fig. 2.) Therefore, we should expect the equilibrium price to decrease as we move away from the origin. For a given price, $\pi < \pi_0$, we can write a system of three equations with unknowns l^1, l^2, l^3 which requires the market to clear the first good at the points $P_1 = (l^1, 0, 0)$, $P_2 = (0, l^2, 0)$, and $P_3 = (0, 0, l^3)$, assuming the price π prevails in the triangle $P_1 P_2 P_3$ and only there. In P_1, for instance, individual 1 knows he is at P_1; individual 2 maximizes his expected utility (subject to his budget constraint) along the segment $P_1 P_3$, and his demand is the average of his demand at P_1 and P_3 (had he been able to tell he was at these points); individual 3, similarly, maximizes his expected utility along the segment $P_1 P_2$.

If we denote by $W(\pi) = w_1 + \pi w_2$ the wealth of each of the individuals at the price π, the equation that requires market clearing (by equating supply and demand) for the first good at P_1 is

$$W(\pi)\left[\alpha^1(l^1, 0, 0) + \frac{\alpha^2(0, 0, l^3) + \alpha^2(l^1, 0, 0)}{2} + \frac{\alpha^3(0, l^2, 0) + \alpha^3(l^1, 0, 0)}{2}\right] = 3w_1,$$

and similarly at the points P_2 and P_3. By walras' law, market clearing for the first good implies market clearing also for the second. Market clearing at the corners, P_1, P_2, P_3, implies market clearing in the entire triangle $(P_1 P_2 P_3)$, as explained above.

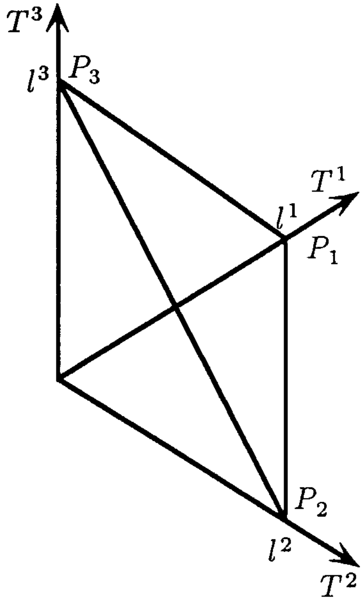

Figure 2

Explicitly, the equations for market clearing for the first good at P_1, P_2, P_3, are

$$W(\pi)\left[(al^1+d)+\left(\frac{bl^3+cl^1}{2}+d\right)+\left(\frac{cl^2+bl^1}{2}+d\right)\right]=3w_1,$$

$$W(\pi)\left[\left(\frac{cl^3+bl^2}{2}+d\right)+(al^2+d)+\left(\frac{bl^1+cl^2}{2}+d\right)\right]=3w_1,$$

$$W(\pi)\left[\left(\frac{bl^2+cl^3}{2}+d\right)+\left(\frac{cl^1+bl^3}{2}+d\right)+(al^3+d)\right]=3w_1.$$

This system is symmetric in l^1, l^2, l^3, and, thus, the solutions, $l^1(\pi)$, $l^2(\pi)$, $l^3(\pi)$, are equal. This common value $l(\pi)$, is the solution to any one of the equations when we replace l^1, l^2 and l^3 by l,

$$W(\pi)[(a+b+c)\,l+3d]=3w_1.$$

Hence,

$$l(\pi)=\frac{3[w_1-W(\pi)\,d]}{W(\pi)(a+b+c)}.$$

For $\pi < \pi_0$, we have $W(\pi) < W(\pi_0)$, and, hence, $l(\pi) > 0$, since, as we checked above, $w_1 - W(\pi_0)\, d = 0$. Furthermore,

$$\frac{dl(\pi)}{d\pi} = \frac{-w_2 d}{W(\pi)(a+b+c)} - \frac{3[w_1 - W(\pi)\, d]}{(a+b+c)\, \pi^2 d},$$

which is negative for $\pi < \pi_0$. This confirms our assumed structure of the equilibrium: Indeed, different prices prevail in different triangles. So, the way individuals compute their posterior beliefs when they observe the price, π, and their private signal, t^i, is indeed the one we assumed. This is because the function $\pi \to l(\pi)$ is one-to-one in the domain. The rational expectations equilibrium price at a point $t = (t^1, t^2, t^3)$ is, therefore,

$$\pi = l^{-1}(t^1 + t^2 + t^3) = \frac{3w_1}{w_2[(a+b+c)(t^1+t^2+t^3) + 3d]} - \frac{w_1}{w_2}. \tag{1}$$

This rational expectations equilibrium is robust to perturbations in the endowments and in the preference parameters[1] of individuals, provided a compensating perturbation in the state space is allowed. With perturbed endowments, (w_1^i, w_2^i) and perturbed preference parameters, a^i, b^i, c^i, d^i, we are still able to solve the economy at the origin, and to find an equilibrium price, π_0, there, and the above system of equations turns out to be

$$W^1(\pi)(a^1 l^1 + d^1) + W^2(\pi)\left(\frac{b^2 l^3 + c^2 l^1}{2} + d^2\right) + W^3(\pi)\left(\frac{c^3 l^2 + b^3 l^1}{2} + d^3\right) = \sum_{i=1}^{3} w_1^i,$$

$$W^1(\pi)\left(\frac{c^1 l^3 + b^1 l^2}{2} + d^1\right) + W^2(\pi)(a^2 l^2 + d^2) + W^3(\pi)\left(\frac{b^3 l^1 + c^3 l^2}{2} + d^3\right) = \sum_{i=1}^{3} w_1^i,$$

$$W^1(\pi)\left(\frac{b^1 l^2 + c^1 l^3}{2} + d^1\right) + W^2(\pi)\left(\frac{c^2 l^1 + b^2 l^3}{2} + d^2\right) + W^3(\pi)(a^3 l^3 + d^3) = \sum_{i=1}^{3} w_1^i,$$

[1] To emphasize, we do not assert robustness with respect to *arbitrary* perturbations of the utility functions.

where $W^i(\pi) = w_1^i + \pi w_2^i$. The system is only nearly symmetric in l^1, l^2, l^3. Since the price domain is compact, for $\{(w_1^i, w_2^i), (a^i, b^i, c^i, d^i): i = 1, 2, 3\}$, in a sufficiently small neighborhood of the original parameters, $dl^i(\pi)/d\pi$ continues to be continuous and negative in the domain $\hat{\pi}_0 \geqslant \pi \geqslant \hat{\pi}_m$, where $\hat{\pi}_m$ is the largest price, π for which

$$\max\{l^1(\pi), l^2(\pi), l^3(\pi)\} = m.$$

In the perturbed economy, the frontier triangle has to be slightly tilted, so the domain of uncertainty for the new economy becomes

$$\left\{(t^1, t^2, t^3): 0 \leqslant \frac{t^1}{l^1(\hat{\pi}_m)} + \frac{t^2}{l^2(\hat{\pi}_m)} + \frac{t^3}{l^3(\hat{\pi}_m)} \leqslant 1\right\}.$$

In this domain, different prices still prevail in different (slightly tilted) triangles. In a given triangle, the demand of individual i continues to be an affine function of his signal, t^i. This means that market clearing at the corners of the triangle, as guaranteed by the new system of equations, implies market clearing in the entire triangle, and hence an equilibrium.

The posterior distribution of individual 1, for instance, when he gets signal $\bar{t}^1$ and observes a price π his is a uniform distribution along the segment whose endpoints are $(\bar{t}^1, l(\pi) - \bar{t}^1, 0)$ and $(\bar{t}^1, 0, l(\pi) - \bar{t}^1)$. Since, by assumption, $b \neq c$, the utility of individual 1 is not constant along this segment:

$$\alpha^1(\bar{t}^1, l(\pi) - \bar{t}^1, 0) = a\bar{t}^1 + b[l(\pi) - \bar{t}^1] \neq a\bar{t}^1 + c[l(\pi) - \bar{t}^1]$$
$$= \alpha^1(\bar{t}^1, 0, l(\pi) - \bar{t}^1).$$

Thus, except for the case $\bar{t}^1 = l(\pi)$, at equilibrium, individual i remains with nontrivial uncertainty regarding his own utility function. Furthermore, along that segment individuals 2 and 3 are demanding different amounts at different points: in $(\bar{t}^1, l(\pi) - \bar{t}^1, 0)$ individual 3 demands $(1/2(b + c)\, l(\pi) + d)\, W(\pi)$ of the first good, while individual 2 demands $[a(l(\pi) - \bar{t}^1) + 1/2(b\bar{t}^1 + c\bar{t}^1) + d]\, W(\pi)$, vice versa in $(\bar{t}^1, 0, l(\pi) - \bar{t}^1)$, and the appropriate weighted means in the points in between. Therefore, at equilibrium individual 1 remains uncertain also regarding the amounts of goods bought or sold by the other two individuals. By symmetry, this is true also for individuals 2 and 3. For sufficiently small perturbations of the parameters of the economy, this sort of asymmetry of information continues to persist also at the rational expectations equilibrium for the perturbed economy described above.

3. CONCLUDING REMARKS

The Volume of Trade as an Additional Public Signal

If the individuals were allowed to observe and extract information also from the total volume of trade, the same equilibrium prices may be maintained. Typically, each individual would then observe at equilibrium a segment or a pair of points symmetric around the midpoint of the interval that he observed before. Thus his posterior utility function would be sustained, and therefore also the equilibrium prices.

More Individuals and Goods

The above may be extended to set ups with more individuals and goods, where N individuals have log-linear utilities in L goods, and where the coefficients of the utilities depend linearly on the state in the N-dimensional space. The $L-1$ equilibrium prices would typically reveal to each individual an $N-L$ dimensional polyhedron, and the actions of the other individuals would be different across the states of each such polyhedron.

The Information Revealed by a Smooth Rational Expectations Equilibrium

In any smooth enough rational expectations equilibrium with L goods and N individuals who get, each, a K-dimensional private signal, each individual will typically remain at equilibrium with a $(N-1)K-(L-1)$-dimensional uncertainty. This will happen at all the regular values of the price function, which, by Sard's theorem are dense with a null complement.

Two Periods of Trade

Similar partial revelation properties prevail in a two-period economy with a real asset and the same state space as in the original example. The utility functions of the individuals are

$$u^i = (x, y_1, y_2) = x(1-x) + y_1(1-y_1) + y_2(1-y_2), \qquad i = 1, 2, 3.$$

In the first stage x trades versus an asset θ that pays one unit of y_1 in each of the states in the second period. Each individual is endowed with amount d of x. The uncertainty regards the second period endowments (w_1^i, w_2^i) of (y_1, y_2), which depend on the state of the world $t = (t^1, t^2, t^3)$ in the following way:

$$\begin{aligned}
w_1^i &= \tfrac{1}{3}(a+b+c)(t^1+t^2+t^3)+d, \qquad i = 1, 2, 3,\\
w_2^1 &= at^1 + bt^2 + ct^3 + d,\\
w_2^2 &= at^2 + bt^3 + ct^1 + d,\\
w_2^3 &= at^3 + bt^1 + ct^2 + d,
\end{aligned}$$

where $a, b, c, d > 0$, $b \neq c$, and $(a+b+c)\,m + 3d < 1/2$ (the last condition guarantees that no individual can accumulate more than 1/2 of each of the goods, so his marginal utility of the goods is always positive).

If y_1 and y_2 always cost the same in the second period, one may compute that there exist first-period equilibrium prices with a structure and properties analogous to those of the example in the previous section.

Trading Constraints

The structure of the set of competitive equilibria when the asset market is incomplete and trading constraints are operative, which reflect the information available to individuals when they decide their demand and supply of commodities, is similar to the structure of the set of equilibria with nominal assets; in particular, competitive equilibrium allocations and prices are indeterminate, as in Polemarchakis and Siconolfi [12]. There exist economies with finitely many states of the world and robust equilibria, partially revealing the actions of individuals. This is due to the degrees of freedom provided by the prices of constrained commodities. Heifetz and Polemarchakis [8] carries out the construction.

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