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Asset markets and the information revealed by prices*

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Summary. When nominal assets serve to transfer revenues across states of the world, noninformative rational expectations equilibria exist. At noninformative prices, the restricted information under which individuals optimize can be modelled as restricted participation of the individuals in asset markets. When assets are nominal, the indeterminacy of equilibrium prices, and, generically, allocations as well, which characterizes economies with restricted participation guarantees that noninformative equilibrium prices exist.

1. Introduction

When information privately available differs across individuals, prices acquire a role in addition to that of conveying the aggregate scarcity of commodities: they convey information across individuals. At a rational expectations equilibrium, individuals refine their private information with the information revealed by prices.

Economies with uncertainty are encompassed by the general equilibrium model of Arrow and Debreu as long as there is a complete market either in elementary securities, Arrow [3], or in contingent commodities, Debreu [9], but not when the asset market is incomplete, Radner [18]. Further, the information available to them may differ across individuals as long as they do not extract information from prices, Radner [17].

First, Radner [19] introduced the informative role of prices in a general equilibrium model with differential information. He showed that, with finitely many signals or states of private information, generically, rational expectations equilibria exist. Furthermore, again generically, all rational expectations equilibria are fully revealing and, therefore, no essential differences of information across individuals persist at equilibrium. Subsequently, Allen [1], [2] and Jordan [12] considered

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infinite signal spaces and focused on the dimension of the signal space relative to that of the domain of prices as the determinant of the existence of fully revealing or approximately fully revealing equilibria.

A question concerning rational expectations equilibria is the characterization of economies in which differences of information persist at equilibrium. Robust examples have been constructed either with the introduction of ad hoc noise, Grossman and Stiglitz [11], or with the introduction of signal spaces of high dimension relative to the dimension of the space of observable relative prices, Ausubel [4].

Here, we demonstrate the existence of noninformative rational expectations equilibria, in which prices convey no information. Rather than looking at particular specifications of the signal space as a source of nonrevelation, we focus, generalizing the example in Mischel, Polemarchakis and Siconolfi [15], on the asset structure as the determinant of the information conveyed by equilibrium prices. In order to focus on the role of the financial structure, we work with finitely many states of private information.

In the literature on rational expectations, it is assumed either that contingent commodities are traded and hence the asset structure is only implicit, Allen [1], [2], or that assets are real, their payoffs are denominated in commodities, Radner [19], Jordan [12]. Here, assets serve to transfer revenue across states of the world and are nominal, their payoffs are denominated in units of account.

We prove that noninformative rational expectations equilibria exist in an economy which extends over two periods, with nominal bonds as the only financial instruments.

This we obtain by exploiting the indeterminancy of equilibrium prices and, generically, allocations as well which characterizes economies with an incomplete asset market and nominal assets, Cass [8], Balasko and Cass [5] and Geanakoplos and Mas-Colell [10].

The degree of indeterminacy suffices to prevent any revelation of information under the assumption that first period endowments of individuals are signal invariant and, more importantly, their preferences over first period consumption are invariant under permutations of first period consumption bundles across information realizations. Indeed, whenever preferences are represented by intertemporally separable, von Neumann–Morgenstren utility functions, as is standard, they satisfy this invariance property, which implies that the only effect of second period prices on first period demand is an income effect. Thus, irrespective of the number of commodities, one degree of indeterminacy suffices to prevent first period commodity prices from revealing any information. First period signal invariance is, evidently, a restrictive assumption.

The introduction of real assets along with the nominal ones does not prevent indeterminacy, Pietra [16], and, hence, it is straightforward to conjecture, neither the existence of noninformative rational expectations equilibria, as shown in Mischel [14] in an example.

We proceed by reducing the existence of noninformative rational expectations equilibria to the existence of competitive equilibria in a particular economy with restricted participation in the asset markets. However, while in the case of nominal assets and unrestricted participation of at least one individual, any nonarbitrage asset prices are competitive equilibrium asset prices, Cass [7], this is not the case with restricted participation, Balasko, Cass and Siconolfi [6]. Thus, we need to develop a complete, formal argument.

The indeterminacy of competitive equilibrium prices and allocations, which allows for the existence of noninformative rational expectations equilibria when assets are nominal, nevertheless, raises a conundrum concerning the definition of a fully revealing rational expectations equilibrium. A fully revealing rational expectations equilibrium is a selection out of the equilibrium correspondence. In our economy, each individual, having received a private signal, observes first period equilibrium prices and forms expectations concerning second period prices. But since equilibria are indeterminate, it is possible to construct robust examples of economies where, for each possible price selection, the first period equilibrium prices are associated with multiple aggregate signal configurations. This does not exclude the possibility of a selection which is one-to-one in signals, but it casts serious doubts on how this selection is to arise as a decentralized market equilibrium. It seems therefore sensible to refine the definition of a fully revealing equilibrium by requiring, not only the classical one-to-one property with aggregate information, but also an incompatibility with other signal configurations. We call an equilibrium which satisfies this further condition a "strong" rational expectations equilibrium. With real assets, this concept is generically equivalent to the standard definition of rational expectations, equilibrium. We construct a robust example of an economy with nominal assets in which strong rational expectations equilibria do not exist.

The existence of noninformative equilibria does not exclude the existence of fully revealing ones. It suffices to observe that the selection of a price normalization yields an economy with real assets, in which, generically, fully revealing equilibria exist.

A characterization of the relative sizes of the sets of fully revealing and not fully revealing, in particular noninformative, rational expectations equilibrium allocations is an interesting open question.

2. The economy

Economic activity occurs over two periods, $t = 1, 2$. Payoff relevant uncertainty is indexed by states of the world or payoff relevant environments, $s \in \mathbf{S} = \{1, \ldots, S\}$, and is resolved in the second period.

Commodities, $c \in \mathbf{C} = \{1, \ldots, C\}$, are traded each period. A commodity bundle is $x = (\ldots, x_c, \ldots)$, a column vector of dimension C.

Assets, $a \in \mathbf{A} = \{1, \ldots, A\}$, are traded in the first period and pay off in the second. A portfolio is $y = (\ldots., y_a, \ldots)$, a column vector of dimension A. Assets payoffs are denominated in units of account. The payoff of asset a in state s is $r_{s,a}$ and R is the matrix of payoffs, of dimension $S \times A$.

Assumption 1: The matrix of asset payoffs, R, has full column rank, A, where $1 \leq A \leq S$.

Individuals are $h \in \mathbf{H} = \{1, \ldots, H\}$. An individual is characterized by his preferences, his initial endowment and his private information. The private information of individual h is described by a finite set of signals, $\mathbf{N}^h = \{1, \ldots, n^h, \ldots, N^h\}$.

The set of joint signals or private information across individuals is $\mathbf{N} = \prod_{h \in \mathbf{H}} \mathbf{N}^h = \{1, \ldots, n, \ldots, N\}$, where $n = (\ldots, n^h, \ldots)$ and $N = \prod_{h \in \mathbf{H}} N^h$. The set of all possible $(H-1)$-tuples of private signals for individuals other than h is $\mathbf{N}^{-h} = \prod_{k \in \mathbf{H} \setminus \{h\}} \mathbf{N}^k$, its cardinality is $N^{-h} = \prod_{k \in \mathbf{H} \setminus \{h\}} N^k$ and $n^{-h} = (\ldots, n^{h-1}, n^{h+1}, \ldots)$ its representative element. The private information of individual h is alternatively described by the partition $\mathscr{L}^h = \{\ldots, \mathbf{L}^h, \ldots\} = \{\mathbf{N}^h \times \{\mathbf{N}^{-h}\}\}$ of the set, $\mathbf{N}$ of joint signals.

Across realizations of private information, a first period commodity bundle is $x_1 = (\ldots, x_1(n), \ldots)$ and a portfolio is $y = (\ldots, y(n), \ldots)$. A signal invariant first period commodity bundle is $x_1 = (\ldots, \hat{x}_1, \ldots)$. A second period commodity bundle for a given realization of private information in the first period is $x_2(n) = (\ldots, x_2(s, n), \ldots)$, and $x_2 = (\ldots, x_2(n), \ldots)$ is a second period commodity bundle across realizations of the state of private information.

A pair (x_1, y) is a function with domain the set of joint signals, $\mathbf{N}$, and thus it is measurable with respect to a partition $\mathscr{L} = \{\ldots, \mathbf{L}, \ldots\}$ of the set $\mathbf{N}$ as long as $(x_1(n), y(n)) = (x_1(n'), y(n'))$ whenever $n, n' \in \mathbf{L}$.

The information available to an individual in the first period need not coincide with his private information. For a partition $\mathscr{L} = \{\ldots, \mathbf{L}, \ldots\}$ of the set $\mathbf{N}$, the preferences of individual h are represented by a utility function, $v^h(\mathscr{L})$, defined over consumption bundles, non-negative commodity bundles, $(x_1, x_2) \geq 0$, such that first period consumption is measurable with respect to the partition $\mathscr{L}$.

The initial endowment of individual h is (w_1^h, w_2^h).

The individual excess demand vector will be denoted by $z^h = (x_1^h - w_1^h, y^h, x_2^h - w_2^h)$, while $z_1^h = (x_1^h - w_1^h)$ and $z_2^h = (x_2^h - w_2^h)$.

Assumption 2: For $h \in \mathbf{H}$,
(i) for any partition $\mathscr{L}$, of the set of joint signals, $\mathbf{N}$, which is at least as fine as the private information partition, $\mathscr{L}^h$, the utility function $v^h(\mathscr{L})$ is continuous, strictly concave and strictly monotonically increasing. Moreover, $v^h(\mathscr{L})$ is invariant under permutations of the first period commodity bundles across information realizations, $v^h(x_1, x_2; \mathscr{L}) = v^h(x_1', x_2; \mathscr{L})$, whenever $x_1(n) = x_1'(n')$, for all $n \in \mathbf{L}$ and $n' \in \mathbf{L}' = \pi(\mathbf{L})$, for some permutation, $\pi: \mathscr{L} \to \mathscr{L}$;
(ii) the initial endowment is strictly positive, $w^h \gg 0$, and the first period endowment is signal invariant, $w_1^h = (\ldots, \hat{w}_1^h, \ldots)$.

The von Neumann–Morgenstern representation of preferences distinguishes between the probability beliefs of the individuals and their cardinal preferences over certain consumption bundles. This distinction is not important for our argument and the more general class of preferences that we consider makes this clear.

Indeed, suppose that conditional on his information in the first period, the preferences of the individual are described by the intertemporally separable von Neumann–Morgenstern utility function defined by $v_1^h(x_1(\mathbf{L})) + E_{(s,n)|\mathbf{S} \times \mathbf{L}} v_2^h(x_2(s, n))$, where $E_{(s,n)|\mathbf{S} \times \mathbf{L}}$ is the expectation over (s, n) conditional on $\mathbf{S} \times \mathbf{L}$. Then, the function over consumption bundles across realizations of first period information defined by $v^h(x_1, x_2, \mathscr{L}) = \sum_{\mathbf{L} \in \mathscr{L}} [v_1^h(x_1(\mathbf{L})) + E_{(s,n)|\mathbf{S} \times \mathbf{L}} v_2^h(x_2(s, n))]$ satisfies first period signal invariance.

First period commodity prices if the joint signal $n \in \mathbf{N}$ is realized are $p_1(n) = (\ldots, p_{1,c}(n), \ldots)$, a strictly positive row vector of dimension C, and asset prices are

$q(n) = (\ldots, q_a(n), \ldots)$, a row vector of dimension A in the domain of nonarbitrage asset prices, $\mathbf{Q} = \{\hat{q} \in \mathbb{R}^A : \hat{q} = \mu R, \text{ for some } \mu \in \mathbb{R}^s_{++}\}$. First period prices are a pair (p_1, q), where $p_1 = (\ldots, p_1(n), \ldots)$ are commodity prices, and $q = (\ldots, q(n), \ldots)$ are asset prices across realizations of private information. Note that (p_1, q) is a function with domain $\mathbf{N}$, on which it induces a partition, the coarset partition with respect to which it is measurable. Second period commodity prices if the state of the world $s \in \mathbf{S}$ and the joint signal $n \in \mathbf{N}$ are realized are $p_2(s, n) = (\ldots, p_{2,c}(s, n), \ldots)$, a strictly positive row vector of dimension C. Second period prices for a given realization of private information in the first period, but across states of the world in the second are $p_2(n) = (\ldots, p_2(s, n), \ldots)$, and $p_2 = (\ldots, p_2(n), \ldots)$ are second period prices across realizations of private information.

Prices are a triple, $p = (p_1, q, p_2)$.

As will be clear from the argument, many of our structural hypotheses can be relaxed. In particular, the strict concavity and strict monotonicity of the utility functions, as opposed to concavity and monotonicity, are not essential. The invariance of asset payoffs with respect to n can also be eliminated, but at the cost of a more complicated argument.

The assumptions which are essential for the argument are the signal invariance of the utility functions over first period consumptions, the signal invariance of the first period endowments and the independence of asset payoffs from second period commodity prices.

At prices (p_1, q, p_2), if the state of private information is n, an individual, after receiving his private signal, n^h, observes first period prices, $(p_1(n), q(n))$, and forms expectations concerning second period prices. Individuals know the dependence of first period prices on the joint signal, which they exploit in order to refine their private information.

At prices p, the partition of $\mathbf{N}$ induced by (p_1, q) is $\mathscr{L}(p) = \{\ldots, \mathbf{L}_p, \ldots\}$, where $\mathbf{L}_p = \{n \in \mathbf{N} : (p_1(n), q(n)) = (p_1(\mathbf{L}_p), q(\mathbf{L}_p)\}$. Having received signal n^h and observing prices $(p_1(n), q(n))$, the individual can rule out signals that do not lie in the set $\mathbf{L}^h_p = \mathbf{L}_p \cap (n^h \times \{\mathbf{N}^{-h}\})$. It follows that the information available to an individual at prices p is $\mathscr{L}^h(p) = \{\ldots, \mathbf{L}^h_p, \ldots\} = \mathscr{L}(p) \wedge \mathscr{L}^h$, the join or coarsest common refinement of the partition induced by first period prices, $\mathscr{L}(p)$, and the partition which describes the private information of the individual, $\mathscr{L}^h = \mathbf{N}^h \times \{\mathbf{N}^{-h}\}$.

If prices are p and $n = (\ldots, n^h, \ldots) \in \mathbf{L}_p$ is the joint signal realized, the budget constraint of the individual is

$$(1) \qquad (p_1(\mathbf{L}_p), q(\mathbf{L}_p))(z_1(\mathbf{L}^h_p), y(\mathbf{L}^h_p)) = 0,$$

$$p_2(n') \otimes z_2(n') = Ry(\mathbf{L}^h_p), \quad n' \in \mathbf{L}^h_p,$$

where $p_2(n') \otimes z_2(n') = (\ldots, p_2(s, n') z_2(s, n'), \ldots)$ is a column vector of dimension S. The individual optimization problem is

$$(2) \qquad \operatorname{Max} v^h(z_1 + w^h_1, z_2 + w^h_2; \mathscr{L}^h(p))$$

$$\text{s.t.}\ (p_1(n), q(n))(z_1(n), y(n)) = 0, \quad n \in \mathbf{N},$$

$$p_2(n) \otimes z_2(n) = Ry(n), \quad n \in \mathbf{N},$$

$$(z_1, y) \text{ is } \mathscr{L}^h(p) - \text{measurable}.$$

A solution to the individual optimization problem $z^h(p) = (z_1^h, y^h, z_2^h)(p)$ exists and is unique.

Note that, the set of solutions to the optimization problem (2) with the utility function defined by

$$v^h(x_1, x_2; \mathscr{L}^h(p)) = \sum_{\mathbf{L}^h(p) \in \mathscr{L}^h(p)} [v^h(x_1(\mathbf{L}_p^h)) + E_{(s,n)|\mathbf{S} \times \mathbf{L}_p^h} v_2^h(x_2(s,n))]$$

coincides with the set of solutions for the intertemporally separable von Neumann–Morgenstern utility function defined by $E_{n|\mathbf{N}}[v^h(x_1(n)) + E_{s|(\mathbf{S},n)} v_2^h(x_2(s,n))]$. This is the case since the individual maximization problem (2) decomposes, for both formulations of the utility function, into a family of optimization problems indexed by elements of the partition $\mathscr{L}^h(p)$,

$$\begin{aligned} &\operatorname{Max} v_1^h(z_1(\mathbf{L}_p^h) + \hat{w}_1^h) + E_{(s,n)|\mathbf{S} \times \mathbf{L}_p^h} v_2^h(z_2(s,n) + w_2^h) \\ &\text{s.t.} \quad (p_1(\mathbf{L}_p), q(\mathbf{L}_p))(z_1(\mathbf{L}_p^h), y(\mathbf{L}_p^h)) = 0, \\ &\qquad p_2(n') \otimes z_2(n') = Ry(\mathbf{L}_p^h), \quad n' \in \mathbf{L}_p^h. \end{aligned}$$

Therefore, in the special case of intertemporally separable von Neumann–Morgenstern preferences, and under the further restriction that a single commodity be traded at each spot market, the only difference between our formulation and that of Radner [19] is in the financial structure, which, in our case consists of nominal, as opposed to real, assets.

Definition 1: Rational expectations prices, p^* are such that

$$\sum_{h \in \mathbf{H}} z^h(p^*) = 0.$$

3. Noninformative rational expectations equilibria

At noninformative prices p, the information available to an individual, if n is the joint signal realized, coincides with his private information, $\mathbf{L}_p^h = n^h \times \{\mathbf{N}^{-h}\}$, and $\mathscr{L}^h(p) = \mathscr{L}^h = \mathbf{N}^h \times \{\mathbf{N}^{-h}\}$.

Definition 2. Prices p are noninformative if and only if

$$\mathscr{L}(p) = \{\mathbf{N}\}.$$

In the "ex-ante" optimization problem, (2), noninformative prices essentially effect a transformation of the portfolio opportunities. More precisely, the reallocations of revenue attainable by an individual, h, are described by the block diagonal matrix diag $(\ldots, R(n), \ldots)$ of dimension $(NS) \times (NA)$, where $R(n) = R$, for all $n \in \mathbf{N}$, and by the portfolio set $\mathbf{Y}^h = \{y : y \text{ is } \mathscr{L}^h\text{-measurable}\}$. The measurability constraints in the individual portfolio sets capture the differences in information across individuals and, also, imply that the economy is one of restricted participation.

In order to study the rational expectations equilibria of our economy, we consider variants of the economy, in which individuals face appropriately specified budget constraints or prices. When confusion may arise, we refer to the economy as specified up to here as "the original economy."

Proposition 1: Noninformative rational expectations equilibrium prices exist.

Proof: We break the argument into two steps.

Step 1: At noninformative prices p, with $(p_1(n), q(n)) = (\hat{p}_1, \hat{q})$, for all $n \in \mathbf{N}$, consider the "modified" budget constraint

$$\sum_{n \in \mathbf{N}} (\hat{p}_1, \hat{q})(z_1(n), y(n)) = 0,$$

$$p_2(n) \otimes z_2(n) = Ry(n), \quad n \in \mathbf{N},$$

$$(z_1, y) \text{ is } \mathscr{L}^h\text{-measurable.}$$

The modified budget constraints differ from the constraints in the individual optimization problem (2), because the first period constraints associated with a realization of the joint signal $n \in \mathbf{N}$ have been replaced by their sum. Evidently, because of the first period signal invariance of endowments and utility functions, the solution to the individual optimization problem under the modified budget constraints, $z^h(p)$, at noninformative prices, p, satisfies $z_1^h(n, p) = \hat{z}_1^h(p)$ for all $n \in \mathbf{N}$. It follows that under the modified budget constraints the individual optimization problem can be rewritten as the "modified" individual optimization problem,

(3)
$$\begin{aligned} &\operatorname{Max} \hat{v}^h(\hat{z}_1 + \hat{w}_1^h, z_2 + w_2^h; \mathscr{L}^h) \\ &\text{s.t.} \quad \hat{p}_1 \hat{z}_1 + (1/N) \sum_{n \in \mathbf{N}} \hat{q} y(n) = 0, \\ &\qquad p_2(n) \otimes z_2(n) = Ry(n), \quad n \in \mathbf{N}, \\ &\qquad y \text{ is } \mathscr{L}^h\text{-measurable,} \end{aligned}$$

where

$$\hat{v}^h(\hat{z}_1 + \hat{w}_1^h, z_2 + w_2^h; \mathscr{L}^h) = v^h(\ldots, \hat{z}_1 + \hat{w}_1^h, \ldots, z_2 + w_2^h; \mathscr{L}^h),$$

for all vectors $(\hat{z}_1 + \hat{w}_1^h, z_2 + w_2^h)$.

A solution to the modified optimization problem, (3), $\hat{z}^h(p) = (\hat{z}_1^h(p), y^h(p), z_2^h(p))$ exists and is unique. It coincides with a solution to the individual optimization problem, (2), at noninformative prices, p, whenever $\hat{q} y^h(n)$ is n-invariant and, a fortiori, if $y^h(n)$ is n-invariant, for all $h \in \mathbf{H}$.

Consider the economy in which individual h solves the modified optimization problem, (3), "the modified economy," for short. Prices for the modified economy are $\hat{p} = (\hat{p}_1, q, p_2)$, with $q = (\ldots, \hat{q}/N, \ldots)$, and competitive equilibrium prices are such that $\sum_{h \in \mathbf{H}} \hat{z}_h(\hat{p}) = 0$.

Lemma 1: Let $\hat{p}^* = (\hat{p}_1^*, q^* p_2^*)$ be a competitive equilibrium price for the modified economy. For $h \in \mathbf{H}$, $y^h(n, \hat{p}^*) = \hat{y}^{*h}$, for all $n \in \mathbf{N}$ and, hence, $p^* = (p_1^*, q^* p_2^*)$, with $p_1^*(n) = \hat{p}_1^*$ for all $n \in \mathbf{N}$, is a noninformative rational expectations equilibrium price for the original economy.

Proof: By the definition of competitive equilibrium prices for the modified economy, for $h \in \mathbf{H}$, $y^{*h}(n) = y^{*h}(n^h, n^{-h}) = -\sum_{k \in \mathbf{H} \setminus \{h\}} y^{*k}(n^h, n^{-h})$, for all $(n^h, n^{-h}) \in \mathbf{N}^h \times \mathbf{N}^{-h}$. But, for all $k \in \mathbf{H} \setminus \{h\}$, the $\mathscr{L}^k$-measurability constraint in the modified maximization problem implies that y^{*k} is n^h-invariant. Therefore, y^{*h} is n^h-invariant and, equivalently, y^{*h} is n-invariant, as desired. ■

The modified economy is a standard two-period economy under uncertainty with restricted participation in the asset market, Balasko, Cass and Siconolfi [6]. Thus, by Lemma 1, the existence of noninformative rational expectations equilibrium prices for the original economy reduces to the existence of competitive equilibrium prices for the modified economy with $q^*(n) = (\ldots, \hat{q}^*/N, \ldots)$ for all $n \in \mathbf{N}$.

In the case of unrestricted participation of at least one individual, the proof of existence of equilibrium entails one individual maximizing under a unique budget constraint. This implies that, when assets are nominal, any nonarbitrage asset prices are competitive equilibrium asset prices for an appropriate choice of commodity prices, Cass [7]. Obviously, if in the present model, one individual were completely informed, $\mathscr{L}^h = \mathbf{N}$, (and therefore the other individuals were completely uninformed, $\mathscr{L}^k = \{\mathbf{N}\}$, $k \neq h$), he would not face any restrictions in the asset market. Any nonarbitrage asset price q, with $q = (\ldots, \hat{q}/N, \ldots)$, would be an equilibrium asset price of the modified economy for some appropriate choice of commodity prices, and therefore would be a noninformative rational expectations equilibrium. This was the substance of the example in Mischel, Polemarchakis and Siconolfi [15].

With restricted participation in the financial market, it is not possible to find an equilibrium commodity prices vector for any arbitrary choice of nonarbitrage asset prices, Balasko, Cass and Siconolfi [6]. Moreover the standard way to prove the existence of equilibria for an economy with restricted participation in the financial market and nominal assets entails the normalization of second period commodity prices spot by spot. But adopting this technique would transform nominal assets into real assets making impossible, generically, the existence of a noninformative equilibrium.

To overcome these problems, we introduce an "augmented economy" in which we expand the individuals' opportunities in terms of both personalized asset prices and less restrictive portfolios. The expansion is decreasing across the natural order of individuals, so that if $h < h', h, h' \in \mathbf{H}$, individual h has both a richer set of personalized asset prices and a bigger set of portfolio holdings than individual h'. We exploit the bigger portfolio sets to generate a correct boundary behavior of the aggregate demand function on a set of prices big enough to guarantee the existence of an equilibrium. The augmented sets of personalized asset prices, together with the particular mappings used in the fixed point argument, developed in the appendix, are used to control the portfolio holdings of the individuals, so that, in equilibrium, they satisfy the measurability constraints of the modified economy. We show that there exist competitive equilibrium prices for the augmented economy which are competitive equilibrium prices for the modified economy with $q^* = (\ldots, \hat{q}^*/N, \ldots)$ and, therefore, noninformative equilibrium prices for the original economy.

Step 2: Now we modify the constraints in the modified optimization problem of individual $h \in \mathbf{H}$ in two respects:

(i) the asset prices q^h, that the individual h faces;

(ii) the measurability constraints on his portfolio holdings, y^h.

We associate with individual $h \in \mathbf{H}$ the information partition $^*\mathscr{L}^h = \prod_{k \in \{h,\dots,H\}} \times \mathbf{N}^k \times \{\prod_{k \in \{1,\dots,h-1\}} \mathbf{N}^k\}$. Further, we associate with each $\hat{q} \in \mathbf{Q}$ a set of nonarbitrage "personalized" asset prices for individual h,

$$\mathbf{Q}^h(\hat{q}) = \{q^h = (\dots, q^h(n), \dots): q^h(n) \in \mathbf{Q}, \text{ for all } n \in \mathbf{N}, q^h \text{ is } {}^{**}\mathscr{L}^h\text{-measurable, and}$$
$$\sum_{n^{-h} \in \mathbf{N}^{-h}} q^h(n^h, n^{-h}) = \hat{q} N^{-h}, \quad \text{for all } n^h \in \mathbf{N}^h\},$$

where $^{**}\mathscr{L}^h = \prod_{k \in \{h+1,\dots,H\}} \mathbf{N}^k \times \{\prod_{k \in \{1,\dots,h\}} \mathbf{N}^k\}$.

At prices $\hat{p} = (\hat{p}_1, \hat{q}, p_2)$ and $q^h \in \mathbf{Q}^h(\hat{q})$, the optimization problem of individual $h \in \mathbf{H}$ is altered to

(4)
$$\operatorname{Max} \hat{v}^h(\hat{z}_1 + \hat{w}_1^h, z_2 + w_2^h, \mathscr{L}^h)$$
$$\text{s.t.} \quad \hat{p}_1 \hat{z}_1 + (1/N) \sum_{n \in \mathbf{N}} q^h(n) y^h(n) = 0,$$
$$p_2(n) \otimes z_2(n) = R y(n), \quad n \in \mathbf{N},$$
$$y \text{ is } {}^*\mathscr{L}^h\text{-measurable.}$$

A solution, $\hat{z}^h(\hat{p}, q^h) = (\hat{z}_1^h, y^h, z_2^h)(\hat{p}, q^h)$, exists and is unique. Note that in the optimization problem (4), individual $h = 1$ does not face measurability constraints on his portfolio holding, while, by the definition of the set $\mathbf{Q}^1(\hat{q})$, his asset prices. q^1, are n^1-invariant. In general, individual h is constrained to satisfy an $(n^1, \dots, n^{h-1})$-invariance condition on his asset holdings, while his asset prices, q^h, are $(n^1, \dots, n^h)$-invariant. The last individual $h = H$ faces the same asset prices and measurability constraints of the "modified" maximization problem. This and the definition of $\mathbf{Q}^h(\hat{q})$ imply that, for all $h \in \mathbf{H}$, the budget set in the modified maximization problem, (3), at prices $(\hat{p}_1, \hat{q}, p_2)$ is contained in the budget set of the augmented economy, (4), at the same commodity prices for all $q^h \in \mathbf{Q}^h(\hat{q})$. Moreover, observe that $\mathscr{L}^h$ is the argument of the utility function, while $^*\mathscr{L}^h$ is the measurability constraint imposed on the portfolio holdings of individual h.

Consider "the augmented economy," in which individual h, $h \in \mathbf{H}$, solves problem (4).

Competitive equilibrium prices for the augmented economy are $\hat{p}^*$, and $q^{*h} \in \mathbf{Q}^h(\hat{q}^*)$, such that $\sum_{h \in \mathbf{H}} \hat{z}^h(\hat{p}^*, q^{*h}) = 0$.

Lemma 2: Competitive equilibrium prices for the augmented economy such that $y^h(\hat{p}^*, q^{*h})$ is $\mathscr{L}^h$-measurable, exist.

We defer the proof of Lemma 2 to the appendix.

Evidently competitive equilibrium prices for the augmented economy such that the portfolio holdings of individual h, $h \in \mathbf{H}$, $y^h(\hat{p}^*, q^{*h})$, satisfy the $\mathscr{L}^h$-measurability constraints, are competitive equilibrium prices for the modified economy and therefore noninformative rational expectations equilibrium prices for the original economy. ■

4. Fully revealing rational expectations equilibria

Rational expectations equilibrium prices are fully revealing if $\mathscr{L}(p^*) = \mathbf{N}$, i.e., if (p_1, q) is injective in n. Therefore, if p^* are fully revealing rational expectations prices, the individual optimization problem (2) reduces to

$$\text{(5)} \qquad \begin{aligned} &\operatorname{Max} v^h(z_1 + w_1^h, z_2 + w_2^h, \mathbf{N}) \\ &\text{s.t.} \quad (p_1^*(n), q^*(n))(z_1(n), y(n)) = 0, \quad n \in \mathbf{N}, \\ &\qquad p_2^*(n) \otimes z_2(n) = Ry(n), \quad n \in \mathbf{N}. \end{aligned}$$

Let $(z^h, y^h)(p^*)$ be the optimal solution to (5), which exists and is unique. Fully revealing rational expectations equilibrium prices, p^*, are such that $\sum_{h \in \mathbf{H}} z^h(p^*) = 0$.

In the special case of time separable, von Neumann–Morgenstern utility functions the individual maximization problem (5) decomposes in a family of maximization problems indexed by the elements $n \in \mathbf{N}$. If p^* are fully revealing rational expectations prices and if n is the signal realization, each individual solves

$$\text{(6)} \qquad \begin{aligned} &\operatorname{Max} v_1^h(z_1(n) + \hat{w}_1^h) + E_{s|\mathbf{S},n} v_2^h(z_2(s,n) + w_2^h(s,n)) \\ &\text{s.t.} \quad (p_1^*(n), q^*(n))(z_1(n), y(n)) = 0, \\ &\qquad p_2^*(n) \oplus z_2(n) = Ry(n). \end{aligned}$$

Let $(z^h, y^h)(p^*)$ be the optimal solution to (7) across realizations of the joint signal, which exists and is unique.

If p^* are fully revealing rational expectations equilibrium prices, then $(p_1^*(n), q^*(n), p_2^*(n))$ are equilibrium prices of an economy where each individual solves the optimization problem (6) with knowledge of n. Therefore, fully revealing rational expectations equilibrium prices can be decomposed in N equilibrium prices $(p_1^*(n), q^*(n), p_2^*(n))$, for $n \in \mathbf{N}$, of standard general equilibrium economies, eventually with incomplete markets, for short, the "n-economies", for $n \in \mathbf{N}$.

Since a fully revealing rational expectations equilibrium is the composition of N standard equilibrium prices for the n-economies, a problem of interpretation arises.

Let $\mathbf{K}(n) = \{(p_1^*(n), q^*(n), p_2^*(n)): (p_1^*(n), q^*(n), p_2^*(n)$ are equilibrium prices for the n-economy$\}$ and let $\mathbf{K}_1(n) = \{(p_1^*(n), q^*(n)): (p_1^*(n), q^*(n), p_2^*(n)) \in \mathbf{K}(n)$, for some $p_2^*(n)\}$.

Fully revealing competitive equilibrium prices are $p^* = (\ldots, p^*(n), \ldots)$ such that $p^*(n) \in \mathbf{K}(n)$, for each $n \in \mathbf{N}$, and, in addition, $(p_1^*(n), q^*(n)) \neq (p_1^*(n'), q^*(n'))$ whenever $n \neq n'$.

Observe that p^* may be a fully revealing equilibrium price even though, for some $n \in \mathbf{N}$, $(p_1^*(n), q^*(n)) \in \bigcup_{n' \in \mathbf{N} \setminus \{n\}} \mathbf{K}_1(n')$.

Of course, by definition, $(p_1^*(n), q^*(n)) \neq (p_1^*(n'), q^*(n'))$, for all $n \neq n'$, and hence knowledge of p^* and observation of first period prices does reveal the joint signal. It is not evident, however, how revelation can be attained in a decentralized framework.

Strongly fully revealing competitive equilibrium prices, p^*, are such that $(p_1^*(n), q^*(n)) \notin \bigcup_{n' \in \mathbf{N} \setminus \{n\}} \mathbf{K}_1(n')$, for all $n \in \mathbf{N}$.

With finitely many signal and real assets, as in Radner [19], the argument for the generic existence of fully revealing equilibria extends immediately to show the generic existence of strongly fully revealing equilibria.

With nominal assets, there always exists a fully revealing rational expectations equilibrium. Its existence is guaranteed by the indeterminacy of asset prices, which allows picking any nonarbitrage asset prices q^* satisfying $q^*(n) \neq q^*(n')$, for all $n \neq n'$, $n \in \mathbf{N}$ and $n' \in \mathbf{N}$. However, with nominal assets, because of the indeterminacy of equilibrium prices when the asset market is incomplete, $A < S$, Balasko and Cass [5], Geanakoplos and Mas-Colell [10], strongly fully revealing equilibria may not exists, and this nonexistence may be robust to perturbations in the parameters of the economy. We construct an example that displays this phenomenon.

The definition of fully revealing rational expectations equilibrium prices as a selection, one-to-one in private information, out of the equilibrium price correspondence is formally correct. The further requirement of strong full revelation is only a matter of interpretation. It is interesting, nevertheless, that if construction of fully revealing selection is adopted, the appropriate choice of normalization is sufficient to yield full revelation even when, after normalization, fully revealing equilibria may fail to exists.

4.1 The nonexistence of strongly fully revealing equilibria: an example

There are two states of the world, $s \in \mathbf{S} = \{1, 2\}$. Two commodities are traded in the first period, $c = 1, 2$, but for simplicity, only the first commodity, $c = 1$, is traded in the second period spot markets. The only available asset, $a = 1$, is a nominally riskless bond, $r_1 = (1, 1)$. There are only two individuals, $h = i, u$, with intertemporally separable von Neumann–Morgenstern utility functions, with indices $v_1^h = \alpha^h \ln(x_{11}) + (x_{12})$, $v_2^h = \beta^h \ln(x_2)$, and initial endowments $w^h = (w_1^h, w_2^h)$. Individual i has access to a private signal $n' \in \mathbf{N} = \{1, 2, 3\}$, while individual u is completely uninformed.

The probability measure μ over $\mathbf{S} \times \mathbf{N}$ is strictly positive, an element of the interior of the 6-dimensional simplex.

For each realization of the signal, the economy is an economy with an incomplete asset market, $1 = A < S = 2$, and nominal assets. Normalizing, we may set all prices equal to 1 except for $p_{1,2}(n) = p_1(n)$ and $p_2(n)$. Let $c \in (0, 1)$ be an arbitrary constant and suppose that, for each realization of the signal, individual i optimizes subject to the budget constraint

$$z_{11}(n) + p_1(n) z_{1,2}(n) + c p_2(1, n) z_2(1, n) + (1 - c) p_2(2, n) z_2(2, n) = 0,$$

while individual u optimizes under the budget constraints

$$\begin{aligned} z_{11}(n) + p_1(n) z_{1,2}(n) + y(n) &= 0 \\ p_2(1, n) z_2(1, n) &= y(n) \\ p_2(2, n) z_2(2, n) &= y(n) \end{aligned}$$

As in Cass [7], one individual optimizes "as if" there were a complete market in contingent commodities with prices $(1, p_1(n), c p_2(1, n), (1 - c) p_2(2, n))$, which amounts to a further normalization. Furthermore, for any $c \in (0, 1)$, the associated equilibrium prices are equilibrium prices of the original economy.

By a straightforward calculation, first period competitive equilibrium prices, $p_1^*(n; c)$, are a bounded, continuous function of the parameter $c \in (0, 1)$ and $\infty > \sup_{c \in (0,1)} p_1^*(n; c) = p_1^+(n) > p_1^-(n) = \inf_{c \in (0,1)} p_1^*(n; c) > 0$. It follows that $\mathbf{K}_1(n) =$

$\{(p^*_{11}(n), p^*_{12}(n), q^*(n)): q > 0$ and $(p^*_{11}(n), p^*_{12}(n)) = \lambda(1, k)$ for $\lambda > 0$ and $k \in [p_1^-(n), p_1^+(n)]\}$. Since $p_1^+(n)$ and $p_1^-(n)$ are continuous functions of the parameters of the economy, there exists an open set of probability measures μ and endowments w^h, $h = i, u$, such that $\mathbf{K}_1(1) \subset \mathbf{K}_1(2) \cup \mathbf{K}_1(3)$ and hence such that no strongly fully revealing equilibria exist.

The failure of existence of strongly fully revealing equilibria is evidently robust.

Finally note that if the asset market were complete, whether asset were nominal, "mixed" or real would be of no consequences and by a straightforward calculation, strongly fully revealing equilibria would exist generically.

5. Conclusion

We have shown that noninformative rational expectations equilibria exist when nominal assets serve to transfer revenue across states of the world.

When assets serve to transfer revenue across states of the world and prices are noninformative, the restricted information under which individuals optimize can be modelled as restricted participation of individuals in the asset markets. When, in addition, assets are nominal, the essential indeterminacy of equilibrium prices, that characterizes economies with restricted participation, guarantees that noninformative equilibrium prices exist.

The first period signal invariance of the utility function plays an important role in the argument, while it is restrictive. With first period signal invariant utility functions, the only effect of second period prices on first period demand is an income effect. Thus, irrespective of the number of commodities, one degree of indeterminacy suffices to prevent first period commodity prices from revealing the joint signal. With utility functions that do not satisfy this restriction, this is no longer true, and the existence of noninformative equilibria is an open question.

Restricted participation also leads to indeterminacy when assets are real but the cardinality of the sets of states of the world or, for that matters, of the set of joint signals is infinite, as we may conjecture from the argument of Mas-Colell [13] for the simpler case of an incomplete asset market. It is thus of interest whether under appropriate cardinality assumptions noninformative rational expectations equilibria exist even with real assets.

As we mentioned earlier, fully revealing equilibria exist along with the noninformative ones. A question that arises, then is the dimension of the set of fully revealing equilibria relative to that of noninformative ones. Evidently the relevant dimensions are those of the sets of equilibrium allocations and not of prices, since nominal distinctions are not of interest. With finitely many commodities, states of the world and joint signals, under regularity assumptions, the set of fully revealing equilibrium allocations is finite of dimension zero, every time the number of assets is equal to the number of states of the world. Under analogous assumptions, the set of noninformative equilibrium allocations is at least equal to 1. Thus, whenever the asset market provides full insurance against the payoff relevant uncertainty, it cannot be argued that full revelation is more likely than nonrevelation. A general characterization of the dimension of the sets of equilibrium allocations is an open interesting question.

Appendix

Proof of Lemma 2: Without any loss of generality, assume that the first A rows of the matrix of yields R corresponding to $s\in\mathbf{A}=\{1,\ldots,A\}\subseteq\mathbf{S}$ are linearly independent and that they form the $A\times A$-identity matrix. Let $\mathbf{Q}'=\{(\hat{q},(q^h,h\in\mathbf{H})): \hat{q}\in\mathbf{Q}$ and $q^h\in\mathbf{Q}^h(\hat{q}), h\in\mathbf{H}\}$ and $\mathbf{P}_1=\{(\hat{p}_1,\hat{q},(q^h,h\in\mathbf{H})\in\mathbb{R}^C_{++}\times\mathbf{Q}': \|(\hat{p}_1,\hat{q},(q^h,h\in\mathbf{H}))\|=1\}$. $\mathbf{P}_1$ is a bounded set and, since $\hat{p}_1\gg 0$, $\mathbf{P}_1$ is homeomorphic to a convex set, and therefore contractible.

More precisely, let $\hat{p}_{1,\setminus 1}$ be the $(C-1)$-dimensional vector obtained from $\hat{p}_1$ by deleting the first component $\hat{p}_{1,1}$. The nonempty, convex and bounded set

$$\mathbf{V}_1=\{(\hat{p}_{1,\setminus 1},\hat{q},(q^h,h\in\mathbf{H})\in\mathbb{R}^{C-1}_{++}\times\mathbf{Q}': \|(\hat{p}_{1,\setminus 1},\hat{q},(q^h,h\in\mathbf{H}))\|<1\}$$

is diffeomorphic to $\mathbf{P}_1$: the smooth map $v_1:\mathbf{V}_1\to\mathbf{P}_1$, $v_1(\hat{p}_{1,\setminus 1},\hat{q},(q^h,h\in\mathbf{H}))=((1-\|(\hat{p}_{1,\setminus 1},\hat{q},(q^h,h\in\mathbf{H}))\|)^{1/2},(\hat{p}_{1,\setminus 1},\hat{q},(q^h,h\in\mathbf{H}))$, defines such a diffeomorphism (the projection of $\mathbf{P}_1$ onto $\mathbf{V}_1$ is the smooth inverse of v_1).

Let $p_2(s,\mathbf{A})=(p_2(s,n),n\in\mathbf{N})$, for $s\in\mathbf{A}$, be a row vector of dimension NC. Let $\mathbf{P}_2(s,\mathbf{A})=\{p_2(s,\mathbf{A}): p_2(s,\mathbf{A})\gg 0$ and $\|p_2(s,\mathbf{A})\|=1\}$, for $s\in\mathbf{A}$, and let $\mathbf{P}_2(s,n)=\{p_2(s,n): p_2(s,n)\gg 0$ and $\|p_2(s,n)\|=1\}$, $s\in\mathbf{S}\setminus\mathbf{A}$, $n\in\mathbf{N}$.

Let $\mathbf{D}=\mathbf{P}_1\times[\prod_{s\in\mathbf{A}}\mathbf{P}_2(s,\mathbf{A})]\times[\prod_{s\in\mathbf{S}\setminus\mathbf{A},n\in\mathbf{N}}\mathbf{P}_2(s,n)]$ and let d be its representative element. $\mathbf{D}$ is nonempty, bounded and convex up to an homeomorphism.

Repeating the procedure utilized to construct the set $\mathbf{V}_1$ and the map v_1, we can construct nonempty, convex and bounded sets $\mathbf{V}_2(s,\mathbf{A})$, $s\in\mathbf{A}$, and $\mathbf{V}_2(s,n)$, $s\in\mathbf{S}\setminus\mathbf{A}$, $n\in\mathbf{N}$, diffeomorphic to the sets $\mathbf{P}_2(s,\mathbf{A})$, $s\in\mathbf{A}$, and $\mathbf{P}_2(s,n)$, $s\in\mathbf{S}\setminus\mathbf{A}$, $n\in\mathbf{N}$, respectively: the smooth maps $v_2(s,\mathbf{A})$, $s\in\mathbf{A}$, and $v_2(s,n)$, $s\in\mathbf{S}\setminus\mathbf{A}$, $n\in\mathbf{N}$, define such diffeomorphisms. Therefore, the nonempty, convex and bounded set $\mathbf{V}=\mathbf{V}_1\times[\prod_{s\in\mathbf{A}}\mathbf{V}_2(s,\mathbf{A})]\times[\prod_{s\in\mathbf{S}\setminus\mathbf{A},n\in\mathbf{N}}\mathbf{V}_2(s,n)]$ is diffeomorphic to $\mathbf{D}$: the smooth map $v=(v_1,(v_2(s,\mathbf{A}),s\in\mathbf{A}),(v_2(s,n),s\in\mathbf{S}\setminus\mathbf{A},n\in\mathbf{N}))$ defines such a diffeomorphism. For notational simplicity, the analysis below is written using the set $\mathbf{D}$ rather than the convex set $\mathbf{V}$. It is easy to check, exploiting the concavity of the function v, that all the assumptions of the Kakutani's fixed point theorem are satisfied if the maps involved were defined using the set $\mathbf{V}$ and the smooth map v.

Let $\hat{z}(d)=\sum_{h\in\mathbf{H}}\hat{z}_h(d)$ be the aggregate excess demand function of the augmented economy.

Claim 1: The excess demand function $\hat{z}$ is a continuous function on the domain $\mathbf{D}$. Moreover for any sequence $(d^k\in\mathbf{D}, k=1,\ldots)$ and $d^*\in Bd(\mathbf{D})$, $\lim_{k\to+\infty}d^k=d^*\Rightarrow\lim_{k\to+\infty}\|\hat{z}(d^k)\|=+\infty$.

Proof: The first part of the claim is obvious.

Let $d^*=(\hat{p}_1^*,\hat{q}^*(q^{*h},h\in\mathbf{H}),(p_2^*(s,\mathbf{A}),s\in\mathbf{A}),(p_2^*(s,n),s\in\mathbf{S}\setminus\mathbf{A},n\in\mathbf{N}))$.

The definition of the set $\mathbf{D}$ and $w_h\gg 0$ imply that $p_2^*(s,n)w_2^h(s,n)>0$, for all $s\in\mathbf{S}\setminus\mathbf{A}$ and $n\in\mathbf{N}$. Therefore, if there exists $h\in\mathbf{H}$ such that $\lim_{k\to+\infty}\hat{p}_1^k/\|(\hat{p}_1^k,q^{k,h})\|>0$ and $p_2^*(s',n')=0$, for some $s'\in\mathbf{A}$ and $n'\in\mathbf{N}$, the claim follows by observing that there exists a portfolio $y^h>0$, compatible with the first and second period budget constraints and generating positive wealth in the second period spot indexed by (s',n').

If $p_2^*(s,n) > 0$, for all $s \in \mathbf{S}$ and $n \in \mathbf{N}$, either $\hat{p}_1^* > 0$ and $(\hat{q}^*, (q^{*h}, h \in \mathbf{H})) \in Bd(\mathbf{Q}')$ or $\hat{p}_1^* = 0$ and $q^{*h} \neq 0$, for some $h \in \mathbf{H}$. In the first instance the claim is trivial. In the second, the claim follows by observing that individual h can pick a portfolio y^h compatible with the first and second period budget constraints and generating positive wealth in the first period spot, $q^{*h} y^h < 0$.

Therefore, suppose that $\hat{p}_1^* = 0$ and $p_2^*(s,n) = 0$, for some $s \in \mathbf{A}$ and $n \in \mathbf{N}$. Then, by the definitions of the sets $\mathbf{Q}'$ and $\mathbf{Q}^H(\hat{q}^*)$, there exists an asset a^* such that $\lim_{k \to +\infty} \hat{q}_{a^*}^k / \| (\hat{p}_1^k, \hat{q}^k) \| \neq 0$ and, by the definition of the sets, $\mathbf{P}_2(s, \mathbf{A})$, $s \in \mathbf{A}$, there exists $n^* \in \mathbf{N}$ such that $p_2^*(s^*, n^*) > 0$, $s^* = a^*$.

Now we show, using an induction argument on $h \in \mathbf{H}$, that the claim is true if there exists $n^{-H} \in \mathbf{N}^{-H}$ such that $p_2^*(s^*, n^{*H}, n^{-H}) = 0$ and, then, that the claim is also true if $p_2^*(s^*, n^{*H}, n^{-H}) > 0$, for all $n^{-H} \in \mathbf{N}^{-H}$.

By the definition of $\mathbf{Q}^1(\hat{q})$, q^{*1} is n^1-invariant, which implies that $q^{*1}(n^1, n^{*-1}) = q^{*1}(n^*)$, for all $n^1 \in \mathbf{N}^1$. If $q_{a^*}^{*1}(n^*) \neq 0$, the claim follows by observing that relative to spot markets indexed by the couple (s,n), $s \in \mathbf{A}$, $n \in \mathbf{N}$, individual $h = 1$ faces a complete asset market (in Arrow securities). If $q_{a^*}^{*1}(n^*) = 0$ and $p_2^*(s^*, n'^1, n^{*-1}) = 0$, for some $n'^1 \in \mathbf{N}^1$, then either

$$\lim_{k \to +\infty} (\| q_{a*}^{k,1}(n^*) \|)/(\| \hat{p}_1^k \|) = +\infty,$$

or there exists a constant K such that

$$\lim_{k \to +\infty} (\| q_{a*}^{k,1}(n^*) \|)/(\| \hat{p}_1^k \|) = K.$$

In the first instance, we can set $y_a^{k,1}(n) = 0$, for all k, $a \neq a^*$ and $n \neq n^*$ and, since $p_2^*(s^*, n^*) > 0$, we can pick a sequence with $y_{a^*}^{k,1}(n^*) = y_{a^*}^1(n^*)$, $k > k^*$, k^* large enough, compatible with the second period budget constraint in the spot (s^*, n^*) and satisfying $q_{a^*}^{k,1}(n^*) y_{a^*}^1(n^*) > 0$, for all $k > k^*$ and $\lim_{k \to +\infty} (q_{a^*}^{k,1}(n^*) y_{a^*}^1(n^*) / (\| \hat{p}_1^k \|) = +\infty$. Then, under this choice, the individual $h = 1$'s first period budget constraint can be rewritten, for $k > k^*$ as

$$(\hat{p}_1^k / \| \hat{p}_1^k \|) \hat{z}_1^k = (\| q_{a^*}^{k,1}(n^*) \| / \| \hat{p}_1^k \|) \| y_{a^*}^1(n^*) \|.$$

Since $(\hat{p}_1^k / \| \hat{p}_1^k \|)$ is bounded above, while the right hand side is positive and unbounded, the claim follows from the strict monotonicity of the utility function.

In the second instance, we can set $y_a^{k,1}(n) = 0$, for all k, $a \neq a^*$ and $n \neq (n'^1, n^{*-1})$, and since $\lim_{k \to +\infty} (\| q_{a^*}^{k,1} \|)/(\| \hat{p}_1^k \|) = K$, we can pick a sequence $y_{a^*}^{k,1}(n'^1, n^{*-1}) = y_{a^*}^1(n'^1, n^{*-1}) > 0$, $k > k^*$, k^* large enough, compatible with the first period budget constraint. Then under this choice, the individual $h = 1$'s second period budget constraint in spot (s^*, n'^1, n^{*-1}) can be rewritten as

$$(p_2^k(s_1^* n'^1, n^{*-1}) / \| p_2^k(s_1^* n'^1, n^{*-1}) \|) \hat{z}_2^k(s_1^* n'^1, n^{*-1}) = y_{a^*}^1(n'^1, n^{*-1}) / \| p_2^k(s_1^* n'^1, n^{*-1}) \|$$

Since $(p_2^k(s_1^* n'^1, n^{*-1}) / \| p_2^k(s_1^* n'^1, n^{*-1}) \|)$ is bounded above, while the right hand side is positive and unbounded, the claim follows from the strict monotonicity of the utility function.

Therefore, suppose that $p_2^*(s^*, n^1, n^{*-1}) > 0$, for all $n^1 \in \mathbf{N}^1$. By the definition of $\mathbf{Q}^2(\hat{q})$, $q^{*2}(n)$ is (n^1, n^2)-invariant, while, by the definition of the individual maximization problem (4), y^2 is $\mathbf{N}^{-1} \times \{\mathbf{N}^1\}$-measurable. If $q_{a^*}^{*2}(n^*) \neq 0$, the claim follows by observing that, since $p_2^*(s^*, n^1, n^{*-1}) > 0$, for all $n^1 \in \mathbf{N}^1$, individual $h = 2$ can pick a portfolio y^2, with $y_a^2(n^1, n^{-1}) = 0$, for all $a \neq a^*$ and $n^{-1} \neq n^{*-1}$, and with $y_{a^*}^2(n^1, n^{*-1})$, $n^1 \in \mathbf{N}^1$, such that $q_{a^*}^{*2}(n^*) \sum_{n^1 \in \mathbf{N}^1} y^2(n^1, n^{*-1}) < 0$, which is compatible with first and second period budget constraints and guarantees positive wealth in the first period budget constraint and therefore the claim.

If $q_{a^*}^*(n^*) = 0$, by the definition of $\mathbf{Q}^2(\hat{q})$, it must be $q_{a^*}^{*2}(n^1, n^2, n^{*3}, \ldots, n^{*H}) = 0$, for all $(n^1, n^2) \in \mathbf{N}^1 \times \mathbf{N}^2$. Therefore, by the same argument used for individual $h = 1$, the claim is ture if $p_2^*(s^*, n^1, n^2, n^{*3}, \ldots, n^{*H}) = 0$, for some $(n^1, n^2) \in \mathbf{N}^1 \times \mathbf{N}^2$. Iterating this argument for all $h < H$, we obtain that the claim is true if $p_2^*(s^*, n^{*H}, n^{-H}) = 0$, for some $n^{-H} \in \mathbf{N}^{-H}$. But then, if $p_2^*(s^*, n^{*H}, n^{-H}) > 0$ for all $n^{-H} \in \mathbf{N}^{-H}$, since $\lim_{k \to +\infty} \hat{q}_a^{k*} / \| (\hat{p}_1^k, \hat{q}^k) \| \neq 0$, the claim follows by the definition of the set $\mathbf{Q}^H(\hat{q})$ and by observing that individual $h = H$ can transfer positive wealth in the first period budget constraint without violating any second period budget constraints, particularly the ones indexed by (s^*, n^{*H}, n^{-H}), $n^{-H} \in \mathbf{N}^{-H}$. ■

Let $\mathbf{1}_T$ be the row vector of 1's of dimension T.
Let $(\bar{p}_1, \bar{q}, (\bar{q}^h, h \in \mathbf{H})) =$
$(1/\| \mathbf{1}_L, \mathbf{1}_S R, (\ldots, (\mathbf{1}_{NS} \operatorname{diag}(\ldots, R, \ldots)), \ldots)) \|)(\mathbf{1}_L, \mathbf{1}_S R, \ldots, (\mathbf{1}_{NS} \operatorname{diag}(\ldots, R, \ldots)), \ldots))$, $\bar{p}_2(s, \mathbf{A}) = (1/\| \mathbf{1}_{NC} \|)\mathbf{1}_{NC}$, for $s \in \mathbf{A}$, $\bar{p}_2(s, n) = (1/\| \mathbf{1}_C \|)\mathbf{1}_C$, for $s \in \mathbf{S} \backslash \mathbf{A}$ and $n \in \mathbf{N}$. For $k > 0$, let $\mathbf{P}_1^k = \{(\hat{p}_1, \hat{q}, (q^h, h \in \mathbf{H})) \in \mathbf{P}_1 : (\hat{p}_1, \hat{q}, (q^h, h \in \mathbf{H})) = (\hat{p}'_1, \hat{q}', (q'^h, h \in \mathbf{H})) + (1/k) \times (\bar{p}, \bar{q}, (\bar{q}^h, h \in \mathbf{H}))$, for some $\hat{p}'_1 \geq 0$ and $(\hat{q}', (q'^h, h \in \mathbf{H})) \in Cl(\mathbf{Q}')\}$,
$\mathbf{P}_2^k(s, \mathbf{A}) = \{p_2(s, \mathbf{A}) \in \mathbf{P}_2(s, \mathbf{A}) : p_2(s, \mathbf{A}) = p'_2(s, \mathbf{A}) + (1/k)\bar{p}_2(s, \mathbf{A})$, for some $p'_2(s, \mathbf{A}) \geq 0\}$, for $s \in \mathbf{A}$ and
$\mathbf{P}_2^k(s, n) = \{p_2(s, n) \in \mathbf{P}_2(s, n) : p_2(s, n) = p'_2(s, n) + (1/k)\bar{p}_2(s, n)$, for some $p'_2(s, n) \geq 0\}$, for $s \in \mathbf{S} \backslash \mathbf{A}$, $n \in \mathbf{N}$.

The set $\mathbf{D}^k = \mathbf{P}_1^k \times [\prod_{s \in \mathbf{A}} \mathbf{P}_2^k(s, \mathbf{A})] \times [\prod_{n \in \mathbf{N}, s \in \mathbf{S} \backslash \mathbf{A}} \mathbf{P}_2^k(s, n)] \subset \mathbf{D}$ is, for all $k = 1, 2, \ldots$, nonempty, compact and convex up to an homeomorphism. By the continuity of the aggregate excess demand function, there exists a nonempty, compact and convex set $\mathbf{F}^k$ such that $\hat{z}(\mathbf{D}^k) \subseteq \mathbf{F}^k$.

Let $(\hat{z}, F^k) = (\hat{z}, F_1^k, (F_2^k(s, \mathbf{A}), s \in \mathbf{A}), (F_2^k(s, n), s \in \mathbf{S} \backslash \mathbf{A}, n \in \mathbf{N})) : \mathbf{D}^k \times \mathbf{F}^k \to \mathbf{D}^k \times \mathbf{F}^k$ be the map defined by $\hat{z} = \hat{z}(d)$,

$$F_1^k = \operatorname{argmax}_{(\hat{p}_1, \hat{q}, (q^h, h \in H)) \in \mathbf{P}_1^k} \hat{p}_1 \hat{z}_1 + (1/N) \textstyle\sum_{n \in \mathbf{N}} (\sum_{h \in \mathbf{H}} q^h(n) y^h(n));$$

$$F_2^k(s, \mathbf{A}) = \operatorname{argmax}_{p_2(s, \mathbf{A}) \in \mathbf{P}_2^k(s, \mathbf{A})} \textstyle\sum_{n \in \mathbf{N}} (p_2(s, n) z_2(s, n)),\ s \in \mathbf{A};$$

$$F_2^k(s, n) = \operatorname{argmax}_{p_2(s, n) \in \mathbf{P}_2^k(s, n)} p_2(s, n) z_2(s, n),\ s \in \mathbf{S} \backslash \mathbf{A},\ n \in \mathbf{N}.$$

The correspondence (z, F^k) is nonempty, compact and convex – valued up to an homeomorphism. It follows from Kakutani's fixed point theorem that there exists a fixed point, $(z^k, d^k) \in \mathbf{D}^k \times \mathbf{F}^k$.

Consider the sequence $(d^k,\ k = 1, \ldots)$. Without loss of generality, $\lim_{k \to +\infty} d^k = d^* \in Cl(\mathbf{D})$.

Claim 2: $d^* \in \mathbf{D}$.

Proof: By the definition of the domain $\mathbf{D}^k$ and the map $(\hat{z}, F^k)$ and by the first period budget constraints of individuals $0 \geq \hat{p}_1^k \hat{z}_1^k + \sum_{n\in\mathbf{N}} (1/N)(\sum_{h\in\mathbf{H}} q^{k,h}(n) y^{k,h}(n)) \geq \bar{p}_1 \hat{z}_1^k + \sum_{n\in\mathbf{N}} (1/N)(\sum_{h\in\mathbf{H}} \bar{q}^h(n) y^{k,h}(n))$, while $\sum_{n\in\mathbf{N}} p_2^k(s,n) z_2^k(s,n) \geq \sum_{n\in\mathbf{N}} \bar{p}_2(s,n) z_2^k(s,n)$, for $s\in\mathbf{A}$, and $p_2^k(s,n) z_2^k(s,n) \geq \bar{p}_2(s,n) z_2^k(s,n)$, $s\in\mathbf{S}\backslash\mathbf{A}$, $n\in\mathbf{N}$.

From the budget constraints of individuals in the second period markets and the definition of $(\bar{q}, (\bar{q}^h, h\in\mathbf{H}))$ we obtain that

$$\sum_{n\in\mathbf{N}} (1/N)\left(\sum_{h\in\mathbf{H}} \bar{q}^h(n) y^{k,h}(n)\right) = (1/\|(\mathbf{1}_L, \mathbf{1}_S R, (\ldots, \mathbf{1}_{NS}\,\mathrm{diag}(\ldots, R, \ldots), \ldots))\|) \times \left(\sum_{n\in\mathbf{N}} (1/N) \sum_{s\in\mathbf{S}} p_2^k(s,n) z_2^k(s,n)\right),$$

while from the definition of $\bar{p}_2(s, \mathbf{A})$, $s\in\mathbf{A}$, and $\bar{p}_2(s,n)$, $s\in\mathbf{S}\backslash\mathbf{A}$, $n\in\mathbf{N}$, and from the preceding inequalities, we obtain that

$$0 \geq \bar{p}_1 \hat{z}_1^k + (1/\|(\mathbf{1}_L, \mathbf{1}_S R, (\ldots, \mathbf{1}_{NS}\,\mathrm{diag}(\ldots, R, \ldots), \ldots))\|) \times \left(\sum_{n\in\mathbf{N}} \sum_{s\in\mathbf{S}} ((1/N) \bar{p}_2(s,n) \hat{z}_2^k(s,n))\right).$$

Since the coefficients in the last inequalities are independent of k and strictly positive, while the individual consumption sets are bounded below, the result follows from the boundary behavior of the aggregate excess demand function. ■

Since $d^*\in\mathbf{D}$, $\lim_{k\to+\infty} \hat{z}^k = \hat{z}^* = \hat{z}(d^*)$ and there exists k' large enough, such that $\hat{z}^*(d^*)$ is a fixed point of the map $(\hat{z}, F^k)$, for all $k > k'$. Moreover, since $d^*\in\mathbf{D}$, it follows, from the definition of the map (z, F_1^k), $k > k'$, that $\hat{z}_1^* = 0$ and, from the definition of $\mathbf{Q}^h(\hat{q}^*)$ and the $^*\mathscr{L}^h$-measurability constraint on portfolio holdings, $h\in\mathbf{H}$, that

$$\sum_{n^h\in\mathbf{N}^h} y^{*h}(n^h, n^{-h}) \quad \text{is } (n^{-h})\text{-invariant}, \quad h\in\mathbf{H}. \tag{7}$$

Equation (7) and the definitions of the map F_1^k, $k > k'$, and of $\mathbf{Q}^h(\hat{q}^*)$, $h\in\mathbf{H}$, imply that $q^* \sum_{h\in\mathbf{H}} \sum_{n\in\mathbf{N}} y^{*h}(n) = 0$ and, therefore, that

$$\sum_{h\in\mathbf{H}} \sum_{n\in\mathbf{N}} y_a^{*h}(n) = \sum_{n\in\mathbf{N}} y_a^*(n) = 0, \quad \text{for all } a = 1, \ldots, A. \tag{8}$$

Since the first A rows of the matrix of yields R corresponding to $s\in\mathbf{A}$ from the $A \times A$-identity matrix, the budget constraints of the individual maximization problem of the augmented economy imply that

$$\sum_{n\in\mathbf{N}} (p_2^*(s,n) z_2^*(s,n), s\in\mathbf{A}) = \sum_{n\in\mathbf{N}} y^*(n). \tag{9}$$

The definitions of the set $\mathbf{P}_2(s, \mathbf{A})$ and of the map $F_2^k(s, \mathbf{A})$, $s\in\mathbf{A}$ and $k > k'$, imply that the sign of $p_2^*(s,n) z_2^*(s,n)$ is invariant for all $n\in\mathbf{N}$, $s\in\mathbf{A}$. But then (8) and (9) imply that $y^*(n) = 0$, for all $n\in\mathbf{N}$.

Since y^{*h} is, for $h > 1$, by the definition of the individual maximization problem (4), n^1-invariant, y^{*1} is n^1-invariant. Consequently, (7) implies that y^{*1} is n-invariant. Moreover, since $y^{*h}(n^2, n^{-2})$, is, by the definition of the individual maximization problem (4), for $h > 2$, and from the previous argument, for $h = 1$, n^2-invariant, y^{*2} is n^2-invariant. Consequently, (7) implies that y^{*2} is n-invariant. Iterating this argument for all $h \in \mathbf{H}$, we get that y^{*h} is n-invariant for all $h \in \mathbf{H}$, and therefore that y^{*h} satisfies the $\mathscr{L}^h$-measurability constraint of the modified individual optimization problem (3).

Finally, the definitions of the sets $\mathbf{P}_2^k(s, \mathbf{A})$, $s \in \mathbf{A}$, and $\mathbf{P}_2^k(s, n)$, $s \in \mathbf{S} \backslash \mathbf{A}$ and $n \in \mathbf{N}$, $k > k'$, and of the maps $F_2^k(s, \mathbf{A})$, $s \in \mathbf{A}$, and $F_2^k(s, n)$, $s \in \mathbf{S} \backslash \mathbf{A}$ and $n \in \mathbf{N}$, together with $y^* = 0$ imply that $z_2^* = 0$. ■

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