The Geneva Papers on Risk and Insurance Theory, Vol. 15, No. 1 (March 1990), 73-79

Non-Informative Rational Expectations Equilibria When Assets are Nominal: An Example*

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ABSTRACT

When assets are nominal, non-informative rational expectations equilibria exist.

1. Introduction

When information privately available differs across individuals, prices acquire a role in addition to that of conveying aggregate scarcities: they convey information. At a rational expectations equilibrium, individuals refine their information with the information revealed by prices.

Radner [1979] first studied the information revealed by prices at a general equilibrium. In this model, before trade occurs, each individuals receives information in the form of a private signal. Under the assumption that the set of signals is finite, generically, rational expectations equilibria exist.[1] Importantly, rational expectations equilibria are, generically, necessarily fully revealing: at a rational expectations equilibrium, difference of information across individuals vanish.

We provide here, in contrast, a robust example of an economy in which at a rational expectations equilibrium prices fail to reveal any information.

The existence of non-informative rational expectations equilibria derives from our specification of the asset structure.[2] In the literature on rational expectations, assets are real: asset payoffs are denominated in commodities; in the economy we study, asset payoffs are denominated in abstract units of account: assets are nominal.

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* This work was supported by the Deutsche Forschungsgemeinschaft, Sonderforschungsbereich 303 and the Gottfried-Wilhelm-Leibniz-Förderpreis during BoWo '89. It was completed while the second author was visiting the Department of Economics of the University of Bonn.

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[1] It is discontinuities in the information revealed as prices vary that limited the result to generic existence.

[2] Thus, our argument is distinct from that developed in Allen [1981] and Jordan [1982], among others, that focuses on the dimension of the domain of signals relative to that of the domain of prices. We maintain the assumption of a finite domain of signals.

As is standard, if not compelling, individuals do not trade in assets prior to receiving their private signals; thus, the asset market is necessarily incomplete.

For rational expectations equilibria to be necessarily fully revealing, it is essential that equilibrium prices associated with distinct realizations of the joint signal be distinct. When assets are real, equilibrium prices are generically locally unique, up to normalization, and vary continuously with the parameters of the economy.[3] When the asset market is incomplete and assets are nominal, generically, competitive equilibria are indeterminate: there exists a non-empty open set of distinct equilibrium prices and excess demands.[4] The indeterminacy of equilibrium prices allows, as we show, for the existence of non-informative rational expectation equilibria.

The existence of non-informative rational expectations equilibria, evidently, does not prevent the existence of informative, even fully revealing rational expectations equilibria. It casts doubt, nevertheless, on the sufficiency of the price mechanism in eliminating differences in information across individuals.

2. The economy

Economic activity extends over two periods, $t = 1,2$. Uncertainty is described by finitely many states of nature, $s \in S = \{1,\ldots,\bar{s}\}$; all uncertainty is realized in the second period according to the probability measure π.

The probability measure π is strictly positive:

$$\pi(s) > 0, \ s \in S.$$

Commodities $l = 1,\ldots,\ell$ are available each period for any realization of uncertainty. A commodity bundle is x.

Assets $a = 1,\ldots,\bar{a}$ are traded in the first period and pay off in the second. Asset payoffs are denominated in abstract units of account. The payoff of asset a in state s is

$$r_a(s), \ a = 1,\ldots, \bar{a}\,, \ s = 1,\ldots,\bar{s}.$$

The asset structure is described by the matrix of asset returns

$$R = \{r_a(s)\}\begin{smallmatrix} s \,=\, 1,\ldots,\bar{s} \\ a \,=\, 1,\ldots,\bar{a} \end{smallmatrix}$$

A portfolio is y.

Individuals are two, $h = i,u$. Individual h is characterized by his intertemporally separable, von Neumann-Morgenstern utility function

$$v^h(1) + Ev^h(2),$$

[3] This was demonstrated in Geanakoplos and Polemarchakis [1986] for an economy with finitely many states of nature; the argument follows essentially from that in Debreu [1970] for an economy with a complete asset market. Recent results in Mas-Colell [1989] demonstrate the failure of local uniqueness in economies with an incomplete market of real assets when the domain of states of nature has the cardinality of the continuum.

[4] This was first noted in Cass [1985] and further developed in Balasko and Cass [1989] and Geanakoplos and Mas-Colell [1989].

defined over consumption bundles, non-negative commodity bundles; and by his initial endowment

$$w^h = (w^h(1), \ldots, w^h(2,s), \ldots).$$

Also, by his private information structure, a partition P^h of the set of states of nature.

For $h = i,u$ and for $t = 1,2$, the cardinal utility index $v^h(t)$ is continuous, strictly monotonically increasing and strictly concave.

For $h = i,u$, the initial endowment is strictly positive:

$$w^h >> 0.$$

These are strong but standard.

Individual u is uninformed:

$$P^u = \{S\}.$$

Individual i is informed:

$$P^i = \{S_\sigma : \sigma = 1, \ldots, \bar{\sigma}\},$$

where

$$S_\sigma \neq \emptyset, \ \sigma = 1, \ldots, \bar{\sigma}.$$

Before trade occurs in the first period, individual i receives a signal, $\sigma = 1, \ldots, \bar{\sigma}$, that informs him of the element of his information partition that contains the actual state of nature.

The joint signal coincides with the signal by the uninformed individuals.

Note that we can write the matrix of asset payoffs as

$$R = \begin{matrix} R(1) \\ \vdots \\ R(\sigma) \\ \vdots \\ R(\bar{\sigma}) \end{matrix}$$

For $\sigma = 1, \ldots, \bar{\sigma}$, the matrix $R(\sigma)$ has full column rank.

Commodity prices are

$$p = (\ldots, p(1,\sigma), \ldots, p(2,s), \ldots).$$

Asset prices are

$$q = (\ldots., q(1,\sigma), \ldots,).$$

Conditional on the signal, asset prices do not allow for arbitrage if and only if

$$R(\sigma)y(1,\sigma) > 0 \Rightarrow q'(1,\sigma)y(1,\sigma) > 0, \ \sigma = 1, \ldots, \bar{\sigma}.$$

The domains of no-arbitrage asset prices are

$$Q(\sigma) = \{q(1,\sigma) : q'(1,\sigma) = \lambda'(\sigma)R(\sigma), \text{ for some } \lambda(\sigma) >> 0\}, \ \sigma = 1, \ldots, \bar{\sigma}.$$

The intersection of the domains of no-arbitrage prices is non-empty:

$$\bigcap_{\sigma=1}^{\bar{\sigma}} Q(\sigma) \neq \emptyset.$$

3. Non-informative rational expectation equilibria

Prices (p,q) are non-informative if and only if

$$p(1) = p(1,\sigma),\ \sigma = 1,\ldots,\bar{\sigma}$$

and

$$q(1) = q(1,\sigma),\ \sigma = 1,\ldots,\bar{\sigma}.$$

At non-informative prices (p,q), the informed individual, i, solves the following optimization problem:

(i)
$$\begin{aligned}&\text{Max } v^i(w^i(1) + z(1,\sigma);1) + E[v^i(w^i(2,s) + z(2,s);2) \mid \sigma]\\ &\text{s.t. } p(2,s)z(2,s) - r(2,s)y(1,\sigma) = 0,\ s \in S_\sigma,\\ &\qquad p(1)z(1,\sigma) + q(1)y(1,\sigma) = 0,\ \sigma = 1,\ldots,\bar{\sigma}.\end{aligned}$$

A solution to the optimization problem of the informed individual is

$$z^i(2,s),\ s = 1,\ldots,\bar{s}$$

and

$$(z^i(1,\sigma),\ y^i(1,\sigma)),\ \sigma = 1,\ldots,\bar{\sigma}.$$

The uninformed individual, u, solves the following problem:

(u)
$$\begin{aligned}&\text{Max } v^u(w^u(1) + z(1);1) + Ev^u(w^u(1,s) + z(2,s);2)\\ &\text{s.t. } p(2,s)z(2,s) - r(2,s)y(1) = 0,\ s = 1,\ldots,\bar{s}\\ &\qquad p(1)z(1) + q(1)y(1) = 0.\end{aligned}$$

A solution to the optimization problem of the uninformed individual is

$$z^u(2,s),\ s = 1,\ldots,\bar{s}$$

and

$$(z^u(1),\ y^u(1)).$$

A non-informative rational expectations equilibrium is a pair of non-informative prices $(\hat{p},\hat{q})$ and associated solutions to the individual optimization problems such that

$$\hat{z}^i(2,s) + \hat{z}^u(2,s) = 0,\ s = 1,\ldots,\bar{s},$$

and

$$(\hat{z}^i(1,\sigma),\hat{y}^i(1,\sigma)) + (\hat{z}(1),\ \hat{y}^u(1)) = 0,\ \sigma = 1,\ldots,\bar{\sigma}.$$

Proposition: A non-informative rational expectation equilibrium exists.

Proof: We break down the argument into several steps.

Step 1: For the informed individual, consider the modified optimization problem

$$\begin{aligned}&\text{Max } E\,v^i(w^i(1) + z(1,\sigma);1) + Ev^i(w^i(2,s) + z(2,s);2)\\ &\text{s.t. } p(2,s)z(2,s) - r(2,s)y(2,\sigma) = 0,\ s \in S_\sigma,\ \sigma = 1,\ldots,\bar{\sigma}.\\ &\qquad \sum_{\sigma=1}^{\bar{\sigma}} \pi(\sigma)[p(1)z(1,\sigma) + q(1)y(1,\sigma)] = 0\end{aligned}$$

It follows from the intertemporal separability of the objective function, the strict concavity and signal-independence of the first period cardinal utility index and the signal-independence of the first period endowment and prices that a solution satisfies

$$\hat{z}^i(1,\sigma) = \hat{z}^i(1),\ \sigma = 1,\dots,\bar{\sigma}.$$

Thus, the modified optimization problem of the informed individual reduces to

(i') $$\text{Max } v^i(w^i(1) + z(1)) + Ev^i(w^i(2,s) + z(2,s);2)$$
$$\text{s.t. } p(2,s)z(2,s) - r(2,s)y(1,\sigma) = 0,\ s \in S_\sigma,\ \sigma = 1,\dots,\bar{\sigma},$$
$$p(1)z(1) + \sum_{\sigma=1}^{\bar{\sigma}} \pi(\sigma)q(1)y(1,\sigma) = 0.$$

Evidently, a solution to the modified optimization problem *(i')* is a solution to the optimization problem *(i)* whenever

$$\hat{y}(1,\sigma) = \hat{y}(1),\ \sigma = 1,\dots,\bar{\sigma}.$$

Step 2 For the uninformed individual, consider the modified optimization problem

(u') $$\text{Max } v^u(w^u(1) + z(1);1) + E\, v^u(w^u(2,s) + z(2,s);2)$$
$$\text{s.t. } p(2,s)z(2,s) - r(2,s)y(1,\sigma) = 0,\ s \in S_\sigma,\ \sigma = 1,\dots,\bar{\sigma},$$
$$p(1)z(1) + \sum_{\sigma=1}^{\bar{\sigma}} \pi(\sigma)q(1)y(1,\sigma),$$
$$y(1) = y(1,\sigma),\ \sigma = 1,\dots,\bar{\sigma}.$$

Evidently, solutions to the modified optimization problem *(u')* and the optimization problem *(u)* coincide.

Step 3 Consider the economy derived from the economy described above after the following modifications:

– Each asset, $a = 1,\dots,\bar{a}$, is replaced by $\bar{\sigma}$ distinct assets, (a,σ), $\sigma = 1,\dots,\bar{\sigma}$, with payoffs.

$$r_{(a,\sigma)}(s) = \begin{cases} r_a(s) & \text{if } s \in \sigma \\ 0 & \text{otherwise.} \end{cases},\quad \sigma = 1,\dots,\bar{\sigma},\ a = 1,\dots,\bar{a}.$$

Note that with

$$R(\sigma) = \{r_{(a,\sigma)}(s)\}_{\substack{s \in \sigma \\ a = 1,\dots,\bar{a}}}$$

we can write the matrix of asset payoffs in block-diagonal form

$$R = \begin{pmatrix} R(1) & & & & \\ & \ddots & & & \\ & & R(\sigma) & & \\ & & & \ddots & \\ & & & & R(\bar{\sigma}) \end{pmatrix}$$

A portfolio is

$$y = (\dots y_{(a,\sigma)},\dots,).$$

– There is no private information. In particular, individual i receives no signal prior to trading in the first period.

– There are constraints in the participation of individuals in the asset market. In particular, individual u optimizes under the constraints that he holds equal amounts of assets (a,σ) and (a,σ') for any index $a = 1,\ldots,\bar{a}$ and any indices $\sigma, \sigma' = 1,\ldots,\bar{\sigma}$.

Commodity prices in the modified economy necessarily have the form of non-informative commodity prices.

Asset prices in the modified economy are

$$q = (\ldots,q_{(a,\sigma)},\ldots).$$

The optimization problems of individuals i and u in the modified economy coincide with (i') and (u'), respectively, whenever

$$q_{(a,\sigma)}(1) = \pi(\sigma)q_a(1) \quad \sigma = 1,\ldots,\bar{\sigma},\ a = 1,\ldots,\bar{a}.$$

Asset prices q do not allow for arbitrage in the modified economy if and only if

$$Ry > 0 \Rightarrow q'y > 0.$$

The domain of no-arbitrage asset prices in the modified economy is

$$Q = \{q\colon q' = \lambda'R, \text{ for some } \lambda >> 0\}.$$

By a standard argument,[5] given asset prices of $\hat{q} \in Q$, there exist commodity prices $\hat{p}$ such that $(\hat{p}, \hat{q})$ are competitive equilibrium prices for the modified economy.

Step 4 In order to establish the existence of a non-informative rational expectations equilibrium, it suffices to establish the existence of a competitive equilibrium for the modified economy such that

(1) $$\hat{q}_{(a,\sigma)}(1) = \pi(\sigma)\hat{q}_a(1), \quad \sigma= 1,\ldots,\bar{\sigma},\ a = 1,\ldots,\bar{a}$$

and

(2) $$\hat{y}^i_{(a,\sigma)} = \hat{y}^i_a,\ \sigma = 1,\ldots,\bar{\sigma},\ a = 1,\ldots,\bar{a}.$$

Since asset prices are chosen parametrically within the no-arbitrage domain, (1) is possible; it suffices to choose

$$\hat{q} = (\ldots\pi(\sigma)\hat{q}(1),\ldots),$$

where

$$\hat{q}(1) \in \bigcap_{0=1}^{\bar{\sigma}} Q(\sigma).$$

Finally, since at a competitive equilibrium

$$\hat{y}^u_{(a,\sigma)}(1) + \hat{y}^i_{(a,\sigma)}(1) = 0, \quad \sigma = 1,\ldots,\bar{\sigma},\ a = 1,\ldots,\bar{a},$$

at a competitive equilibrium (2) follows.

This complete the argument.■

Remark: The intertemporal separability of the individual utility functions is essential to our argument. It insulates, in the modified optimization problems, first period consumption from the signal received by the informed individual. Also important for our example is the

[5] For example, the argument in Cass [1984].

simple diversity of the economy, in particular, the presence of only one informed individual. In conjunction with the market clearing conditions, it insulates, in the modified economy, the portfolio held by the uniformed individual at equilibrium essentialy from the signal this individual received.

4. Conclusion

The existence of non-informative rational expectations equilibria when assets are nominal should extend to a more general setting.

Monetary policy may serve to limit, even eliminate the indeterminacy of equilibria when assets are nominal. Thus, it may enhance the ability of the price mechanism to convey information.

NOTATION

- All vectors are colum vectors; " ′ " denotes the transpose, also for matrices.
- " $\gg$ ", " $>$ ", " $\geqq$ ": vector inequalities; for scalars, " $\gg$ " and " $>$ " coincide.

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