European Economic Review 33 (1989) 59–65. North-Holland

INTERTEMPORAL EQUILIBRIUM AND DISADVANTAGEOUS GROWTH*

M.P. DONSIMONI

Université Catholique de Louvain, 1348 Louvain-la-Neuve, Belgium

H.M. POLEMARCHAKIS

Columbia University, New York, NY 10027, USA

Received March 1986, final version eceived August 1987

Growth may be disadvantageous in a single competitive economy of overlapping generations with production and investment, in spite of Walrasian stability.

1. Introduction

Growth is disadvantageous when an individual or a country whose endowment or productivity increases suffers a loss in utility. The mechanism which allows for this phenomenon is the change in relative prices or terms of trade at equilibrium.

The striking possibility that the disadvantageous change in relative prices may dominate the advantageous change in the endowment was first noticed in the framework of international trade:

> 'Mill was aware (1894, pp. 150–153) that an increase in productivity would lower the commodity terms of trade and even the factorial terms of trade if foreign demand, in the latter case, were inelastic. Edgeworth interpreted (1925, p. 10) Mill's passage as indicating that a country could be *damnified* by growth, supplying the necessary assumptions to make Mill's analysis correct.'
>
> Mundell (1968; p. 263); emphasis added.

*This work was supported in part by grant no. SES 84-11149 of the National Science Foundation, and by grant no. 2-4514-85 of the Fonds de la Recherche Fondamentale Collective, Belgium. It was carried out while the second author was a visitor at C.O.R.E., Université Catholique de Louvain.

Interest in the possibility of disadvantageous growth was revived with Hicks' (1953) paper on the dollar shortage; there, Hicks advanced the proposition that, in a two-country model with one country growing and the other not, technological progress in the growing country could turn the terms of trade against it if the progress were export-biased. Findlay and Grubert (1959) provided a complete characterization of the variation in the terms of trade.

A disadvantageous change in the terms of trade is, however, only necessary for disadvantageous growth. The first derivation of the conditions for disadvantageous growth were carried out by Meade (1951). Their importance was brought out by Bhagwati (1958), who coined the term 'immiserizing growth' [Corden (1956), and Johnson (1954, 1955)].

In the works cited above, the argument was developed in partial equilibrium: the change in relative prices was specified exogenously and was not derived from market clearing after the change in endowments. Aumann and Peleg (1974) noted this possibility at the general equilibrium of a pure exchange economy.

As long as competitive equilibrium allocations are Pareto optimal, disadvantageous growth is a statement about the distribution of utility gains and losses: in an economy extending over the finite time, with no externalities or distortions, and with complete asset markets for the allocation of risk, growth necessarily leads to a gain in utility for some individual. The point is that, as we have shown elsewhere – Donsimoni and Polemarchakis (1985) – the individuals whose endowment increases need not be the ones who gain in utility.

In this paper we study the possibility of disadvantageous growth at the intertemporal equilibrium of an economy characterized by a structure of overlapping generations with production and investment, as in Diamond (1965). As is well known, in such an economy competitive equilibrium allocations are not necessarily Pareto optimal: the stationary competitive allocation need not satisfy the golden rule. We show that away from the golden rule growth may be disadvantageous for a single country, in the absence of intra-generational heterogeneity, under competition, in the absence of externalities or distortions, with complete markets, and in spite of Walrasian stability.

We do comparative statics: we compare stationary competitive equilibria corresponding to different endowment levels of a resource, and we do not consider the distribution of utility gains and losses on the transition path across stationary equilibria.

The mechanism which allows for disadvantageous growth is the change in the rate of interest at equilibrium. When the endowment of an inelastically supplied factor of production or resource changes, the rate of interest changes to maintain market clearing. The magnitude and direction of the change in the rate of interest depend on the technology and the propensity of individuals to save. Producers as welll as consumer-workers behave competitively, that is parametrically towards prices. The change in the rate of interest may have an adverse effect on the utility attained by individuals; this adverse effect may be strong enough to outweigh the beneficial effect of growth. Growth may thus be disadvantageous.

2. The economy

Economic activity extends over infinite discrete time under certainty. During each period, two generations are alive, one in the first (1) and one in the second (2) period of its life. Generations are identical across time and homogeneous: they consist of a single individual.

An individual is characterized by a smooth intertemporal utility function $u(c_1, c_2)$ defined over positive first and second period consumption, and by an endowment $\bar{x}$ of a resource during the first period of his life. The resource can be interpreted either as labor or as a non-producible natural resource. We suppose with no loss of generality that the individual has no endowment of the resource during the second period of his life.

Production occurs within a period according to the constant returns to scale, smooth, neoclassical production function $f(k,x)$; production possibilities are independent of time. The capital input to production is the output produced but not consumed in the preceding period; for simplicity, the rate of depreciation is one.

At each period, the output is numeraire. The prices of the capital and resource factors are r and w, repectively.

At prices (r, w), the individual in the first period of his life supplies his endowment $\bar{x}$ of the resource inelastically and chooses his first period consumption c_1 and savings s so as to maximize his intertemporal utility subject to the budget constraint. Equivalently, he solves

$$\begin{aligned} &\max u(c_1, c_2) \\ &\text{s.t. } c_1 + s \leqq w\bar{x}, \\ &\qquad c_2 \leqq rs. \end{aligned} \tag{1}$$

A solution yields a consumption plan $(c_1(r, w; \bar{x}), c_2(r, w; \bar{x}))$ and savings $s(r, w; \bar{x})$; the latter consttute the supply of capital by the individual in the second period of his life.

We restrict our attention to points $(r, w; \bar{x})$ at which the behavior of the individual is regular. In particular, the savings function $s(r, w; \bar{x})$ is well defined, positive, continuously differentiable, and satisfies the Slutzky equation

$$s_r = \frac{1}{r^2}(\sigma - mrs), \tag{2}$$

where σ is the non-negative cross-substitution effect between first and second period consumption, and m is the marginal propensity to consume in the first period:

$$s_{\bar{x}} = w(1-m); \qquad s_w = \bar{x}(1-m). \tag{3}$$

Let $\tilde{u}(r, w; \bar{x})$ be the indirect utility function; because of the assumption of regularity, the function $\tilde{u}$ is well defined and continuously differentiable. Without loss of generality, we take the marginal utility of income to be one; it follows from the envelope theorem that

$$\mathrm{d}\tilde{u} = (s/r)\,\mathrm{d}r + \bar{x}\,\mathrm{d}w + w\,\mathrm{d}\bar{x}. \tag{4}$$

The first two terms are the relative price component of the change in utility; the third term is the revenue component.

At prices (r, w), the demand for factors of production is chosen to maximize profit; equivalently, to solve

$$\max f(k, x) - rk - wx. \tag{5}$$

We restrict our attention to points (r, w) at which the behavior of the producer is regular. In particular, $f_k > 0$, $f_x > 0$, and

$$f_{kx} = -(\bar{x}/k) f_{kk} > 0. \tag{6}$$

The assumption of constant returns to scale prevents the derivation of factor demands as functions of prices alone. Since the aggregate supply of the resource $\bar{x}$ is inelastic, it can be substituted into the first order necessary and sufficient marginal productivity conditions to yield a well defined demand function for the elastically supplied capital and, subsequently, market clearing prices for the inelastically supplied resource. Consider, first, the capital equation for profit maximization

$$f_k(k, \bar{x}) = r. \tag{7}$$

Since $f_{kk} < 0$, the function can be inverted to yield the capital demand function $k(r; \bar{x})$ which is positive, continuously differentiable, and strictly monotonically decreasing:

$$k_r = f_{kk}^{-1}; \qquad k_{\bar{x}} = -f_{kx}/f_{kk} = k/\bar{x}. \tag{8}$$

Consider, next, market clearing in the market for the resource. With the demand function for capital which we just derived, profit maximization yields the resource price $w(r)$ necessary and sufficient for the resource market to clear:

$$f_x(k(r;\bar{x}),\bar{x}) = w(r). \tag{9}$$

The function $w(r)$, which describes the traditional factor price frontier, is continuously differentiable, positive and strictly monotonically decreasing:

$$w_r = f_{xk}k_r < 0. \tag{10}$$

For a given endowment of the resource $\bar{x}$, a stationary competitive equilibrium price vector $(r(\bar{x}), w(\bar{x}); \bar{x})$ is such that all markets clear:

$$s(r, w(r); \bar{x}) = k(r; \bar{x}); \tag{11}$$

note that the resource market clears if and only if the capital market clears. Totally differentiating the equilibrium condition we obtain

$$(1 - s_r f_{kk} - s_w f_{kx})\,\mathrm{d}r = f_{kk}(s_{\bar{x}} - (k/\bar{x}))\,\mathrm{d}\bar{x}. \tag{12}$$

Local Walrasian stability of an equilibrium reduces to local stability of the dynamical system

$$\dot{r} = k(r) - s(r, w(r)); \tag{13}$$

we have suppressed the dependence on the endowment of the resource since, in the stability argument, its level is held fixed. We restrict our attention to stationary competitive equilibria which are locally Walrasian stable: the behavior of consumers and producers is regular and, in addition,

$$f_{kk}s_r + f_{kx}s_w - 1 < 0. \tag{14}$$

We can now determine the infinitesimal change in prices at a regular stationary competitive equilibrium following an infinitesimal change in the endowment of the resource. Substituting from (2) and (3) into (14) and letting

$$H = kf_{kk}\left[\frac{\sigma}{kf_k^2} - 1 + m\left(1 - \frac{1}{f_k}\right)\right], \tag{15}$$

we obtain that the equilibrium is locally Walrasian stable if and only if[1]

[1]An alternative notion of stability, local dynamic stability, refers to the dynamical systems $k_{t+1} = s(r_t)$, where $r_t = f_k(k_t)$. It reduces to the requirement that $|H| < 1$ and does not alter the argument.

$$1-H>0. \tag{16}$$

Substituting from (2) and (3) into (12), we obtain that

$$\frac{\mathrm{d}r}{\mathrm{d}\bar{x}}=\frac{f_{kk}[f_x(1-m)-(k/\bar{x})]}{1-H}. \tag{17}$$

Substituting into (4) and simplifying we obtain that

$$\frac{\mathrm{d}\tilde{u}}{\mathrm{d}\bar{x}}=f_x-k\left(1-\frac{1}{f_k}\right)\frac{f_{kk}(1-m)-(k/\bar{x})]}{1-H}. \tag{18}$$

Observe that even under the assumption of local Walrasian stability, $1-H>0$, an increase in the endowment of the resource, $\mathrm{d}\bar{x}>0$, may lead to either an increase or a decrease in the price of the resource at equilibrium:

$$\frac{\mathrm{d}w}{\mathrm{d}\bar{x}}=\frac{f_{xk}}{f_{kk}}\frac{\mathrm{d}r}{\mathrm{d}\bar{x}}>0\Leftrightarrow\frac{\mathrm{d}r}{\mathrm{d}\bar{x}}<0\Leftrightarrow s_{\bar{x}}=f_x(1-m)>\frac{k}{\bar{x}}. \tag{19}$$

A stationary competitive equilibrium coincides with the golden rule if and only if $f_k=1$; at the golden rule $(\mathrm{d}\tilde{u}/\mathrm{d}\bar{x})=f_x>0$ and hence growth is always advantageous.

3. Disadvantageous growth

By definition, growth is disadvantageous if and only if

$$\frac{\mathrm{d}\tilde{u}}{\mathrm{d}\bar{x}}<0. \tag{20}$$

Proposition. At a locally Walrasian stable, stationary competitive equilibrium which is characterized by either overinvestment ($f_k<1$) or underinvestment ($f_k>1$), growth may be disadvantageous.

Proof. To establish that growth may be disadvantageous, we may choose arbitrarily the preference characteristics (σ, m), the production characteristics (f_k, f_x, f_{kk}), the endowment level of the resource $\bar{x}$, and the equilibrium level of the capital stock k, subject only to sign restrictions.

Let $f_x=1, k=1$.

Let $f_k=1+\varepsilon$, where ε is small in absolute value but non-zero.

For $\sigma=f_k^2[1-(\frac{1}{2})(1-1/f_k)], 0<\sigma<1$; setting $m=\frac{1}{2}$ yields, from (15), that $H=0$ and hence $1-H>0$.

Substituting into (18) we obtain that

$$\frac{\mathrm{d}\tilde{u}}{\mathrm{d}\bar{x}}=\left(1-\frac{1}{1+\varepsilon}\right)f_{kk}\left(\frac{1}{2}-\frac{1}{\bar{x}}\right).$$

For $\varepsilon<0$, $(1-1/(1+\varepsilon))<0$ and hence setting $\bar{x}>2$, and $f_{kk}<0$ sufficiently large in absolute value yields $(\mathrm{d}\tilde{u}/\mathrm{d}\bar{x})<0$.

Alternatively, for $\varepsilon>0$, $(1-1/(1+\varepsilon))>0$ and hence setting $\bar{x}<2$, and $f_{kk}<0$ sufficiently large in absolute value yields $(\mathrm{d}\tilde{u}/\mathrm{d}\bar{x})<0$. Q.E.D.

A concluding remark is in order: It is necessary, even though not sufficient, for growth to be disadvantageous that either the economy displays over-investment ($f_k<1$) and an increase in the endowment of the resource leads to a decrease in the rate of interest ($\mathrm{d}r/\mathrm{d}\bar{x}<0$) and an increase in the price of the resources ($\mathrm{d}w/\mathrm{d}\bar{x}>0$) at equilibrium, or the economy displays under-investment and the compaarative statics at equilibrium are reversed:

$$\frac{\mathrm{d}\tilde{u}}{\mathrm{d}\bar{x}}=f_x-k\left(1-\frac{1}{f_k}\right)\frac{\mathrm{d}r}{\mathrm{d}\bar{x}}. \tag{21}$$

References

Aumann, R.J. and B. Peleg, 1974, A note on Gale's example, Journal of Mathematical Economics 1, 209–211.

Bhagwati, J., 1958, Immiserizing growth: A geometrical note, Review of Economic Studies 25, 201–205.

Corden, M., 1958. Economic expansion and international trade: A geometrical approach, Oxford Economic Papers 8.

Diamond, P.A., 1985, National debt in a neoclassical growth model, American Economic Review 55, 1126–1150.

Donsimoni, M.P. and M. Polemarchakis, 1985, Variations in endowments and utilities, Discussion paper no. 85 (C.O.R.E., Université Catholique de Louvain, Louvain-la-Neuve).

Edgeworth, F.Y., 1925, Papers relating to political economy, Vol. 2 (Macmillan, New York).

Findlay, R. and H. Grubert, 1959, Factor intensities, technological progress, and the terms of trade, Oxford Economic Papers 11, 111–121.

Johnson, H., 1954, Increasing productivity, income-price trends, and the trade balance, Economic Journal 64, 482–485.

Johnson, H., 1955, Economic expansion and international trade, Manchester School of Economic and Social Studies 23, 95–112.

Hicks, J.R., 1953, An inaugural lecture, Oxford Economic Papers 5.

Meade, J.E., 1951, The balance of payments: Mathematical supplement (Oxford University Press, Oxford).

Mill, J.S., 1894, Principles of political economy, Vol. 2.

Mundell, R.A., 1968, International economics (Macmillan, New York).