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Intertemporal Equilibrium and the Transfer Paradox

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The transfer paradox may occur in a world with only two countries at a dynamically stable intertemporal competitive equilibrium. In a framework of overlapping generations with production and investment, a transfer of income may immiserize the recipient while enriching the donor. Away from the golden rule, a transfer may result in a Pareto improvement.

1. INTRODUCTION

The transfer paradox may occur in a world with only two countries at a dynamically stable intertemporal competitive equilibrium. In a framework of overlapping generations with production and investment, we show that a transfer of income may immiserize the recipient and enrich the donor, while, away from the golden rule, it may result in a Pareto improvement.

It was Leontief's (1936) original observation in the framework of a finite exchange economy with two countries (individuals) that the distribution of utility gains and losses resulting from a transfer may be perverse due to the change in the terms of trade (relative prices). The relevance of this transfer paradox was subsequently challenged by Samuelson (1947) who argued that the paradox cannot occur in this framework unless the equilibrium is Walrasian-unstable.[1] Samuelson's argument thus diverted the attention of the trade-theoretic literature from the welfare aspects of the transfer problem to the analysis of the impact on the terms of trade alone.[2]

Recently, however, the welfare aspects of the transfer problem have received significant attention from international trade theorists as well as from mathematical economists.[3] Two branches of literature have attempted to re-establish the relevance of the transfer paradox. Each relaxes a different assumption necessary for the association of the paradox with instability: one incorporates a third country (individual) into the economy; the other addresses the problem in the presence of distortions. In either case an alternative possibility arises: a transfer may increase (decrease) the welfare of the donor as well as of the recipient.[4]

Gale (1974) constructed an example of a fixed-coefficient, three-agent exchange economy in which a transfer between two agents benefits the donor along with the recipient at a Walrasian-stable equilibrium.[5] In Gale's framework, Chichilnisky (1980) demonstrated that the transfer paradox as well can occur at a stable equilibrium.[6] Independently, the phenomenon was noted by Brecher and Bhagwati (1981) and Yano (1981). Bhagwati, Brecher, and Hatta (1983a) showed that, in a general two-good, three-country framework, a bilateral transfer may immiserize the recipient, enrich the donor, or benefit both, all in a Walrasian-stable equilibrium; in addition, they illuminated the reasoning underlying the phenomenon.[7]

Brecher and Bhagwati (1982) and Bhagwati, Brecher and Hatta (1985) demonstrated that in the presence of an exogenous, policy imposed, or an endogenous, transfer-induced distortion, the transfer paradox is compatible with the stability of equilibrium even in a two-country framework.

We establish that the transfer paradox or the yet unexplored phenomenon of a Pareto improving transfer may occur in a perfectly competitive world (where individual agents are well behaved and fully informed, markets are complete, and externalities or distortions are absent), with only two countries, and in spite of stability. The conditions for the occurrence of the transfer paradox and the phenomenon of a Pareto improving transfer are examined in an intertemporal setting wherein each of the two economies is characterized, along the lines of Diamond (1965), by a structure of overlapping generations with production and investment. The distinguishing feature of the overlapping generation setting is that economic activity extends over infinite time while the planning horizon of every agent is finite; this breaks down the identification of competitive equilibria with Pareto optima.[8]

We focus our analysis on comparative statics across stationary equilibria; we compare the stationary level of utility attained by individuals before and after the transfer. We believe that this is in the tradition of the literature on the transfer paradox, which ignores the distribution of utility gains and losses on the transition path from one equilibrium to the other.

The analysis indicates that, if differences in time preferences, and hence propensities to save, are sufficiently large across countries, and if certain technological requirements are satisfied, the transfer paradox or the phenomenon of a Pareto improving transfers obtain at a stable stationary equilibrium; for the transfer paradox, the latter may even coincide with the golden rule.[9]

2. THE ECONOMY

Economic activity extends over infinite discrete time under certainty and occurs in two distinct countries $i = A, B$. Three goods are available at each period: two factors, inputs to production, capital, labour, and a single output. The prices of factors are r and w, and their quantities k and l, respectively; the output is numeraire, and its quantity is c. A price system is a vector $p = (r, w)$. The endowment of labour at each period is exogenous. The capital input to production is the output produced but not consumed in the preceding period; the rate of depreciation is equal to one.

During each period, two individuals (aggregate consumer-worker-investors) are alive in each country, one in the first and one in the second period of his life, 1 and 2, respectively; there is not growth of population. Within each country, generations are identical across time and homogeneous; individuals are thus also $i = A, B$. An individual i is characterized by the intertemporal utility function $u^i(c_1, c_2)$ defined over consumption during the first and second periods of his life and the endowment $\bar{l}^i > 0$ of labour during the first period of his life; an individual has no endowment during the second period of his life.

Production occurs within a period according to the neoclassical, constant returns to scale production function $f(k, l)$; production possibilities are indpendent of time and invariant across countries. Unlike labour, capital is a mobile factor. It follows that factor prices are equalized and production can be aggregated across countries.

Transfers of income occur across countries. The transfer to country i or, equivalently, to the individual in country i during the first period of this life is t^i; $t^A + t^B = 0$. A country i is a donor if $t^i < 0$ and a recipient if $t^i > 0$. We set $t^A = t$ and $t^B = -t$; for $t > 0$, A is the recipient and B the donor.

At prices $p = (r, w)$ and transfer t, the individual at the first period of his life in country i supplies the endowment of labour $\bar{l}^i$ inelastically and decides on the first period consumption c_1^i and savings s^i so as to maximize the utility function $u^i(c_1^i, c_2^i)$; $c_2^i = rs^i$, while the budget constraint is $c_1^i + s^i \leqq y^i$, where $y^i = w\bar{l}^i + t^i$; equivalently, the individual solves

$$\begin{aligned} &\text{Max}_{c_1, c_2}\, u^i(c_1^i, c_2^i) \\ &\text{s.t.} \quad c_1^i + \frac{1}{r} c_2^i \leqq w\bar{l}^i + t^i. \end{aligned} \tag{1}$$

The solution to (1) is $(c_1^i(p; t^i), c_2^i(p; t^i))$. The savings implied by the solution to (1) are $s^i(p; t^i) = w\bar{l}^i + t^i - c_1^i(p; t^i)$; they constitute the supply of capital by the individual i at the second period of his life, k^i. The level of utility attained is $\tilde{u}^i(p; t^i) = u^i(c_1^i(p; t^i), c_2^i(p; t^i))$.

At prices $p = (r, w)$, the demand for factors of production is chosen so as to maximize profits; equivalently, to solve

$$\text{Max}_{k,l}\, f(k, l) - rk - wl. \tag{2}$$

We restrict our attention to points (p, t) on a neighbourhood of which

(i) a solution to the individual optimization problem (1) exists, is unique, and satisfies

$$c_1^i + s^i = w\bar{l}^i + t^i; \tag{3}$$

(ii) the savings function $s^i(p; t^i)$ is continuously differentiable and satisfies the Slutzky equation

$$s_r^i = \frac{1}{r^2}(\sigma^i - v^i rs^i), \tag{4}$$

where the cross-substitution effect σ^i is non-negative, and v^i is the income effect

$$s_w^i = \bar{l}^i(1 - v^i); \tag{5}$$

$$s_{t^i}^i = 1 - v^i; \tag{6}$$

(iii) the indirect utility function $\tilde{u}^i(p; t^i)$ is continuously differentiable and satisfies

$$d\tilde{u}^i = \left(\frac{1}{r} s^i, \bar{l}^i\right) dp + dt^i; \tag{7}$$

without loss of generality, the marginal utility of income is taken to equal one.

Note that (7) decomposes the change in utility (real income) into a relative price effect and an income effect.

Remark 1. At a point (p, t), the behaviour of individual i is fully described by the vector (s^i, σ^i, v^i) which satisfies the domain restrictions $s^i > 0, \sigma^i \geqq 0$. These restrictions are not only necessary (they follow from standard assumptions about the utility function), but sufficient as well: given a vector $(\bar{s}^i, \bar{\sigma}^i, \bar{v}^i)$ which satisfies the (strong) domain restrictions $\bar{s}^i > 0, \bar{\sigma}^i > 0$, there exists a (twice continuously differentiable, strictly monotone and strictly quasi-concave) utility function whose maximization subject to the budget constraint in a neighborhood of (p, t) yields a consumption and savings function which, at (p, t), satisfies (3), (4), (5) and (6) with $s^i = \bar{s}^i, \bar{\sigma}^i = \bar{\sigma}^i$ and $v^i = \bar{v}^i$. Thus we may specify freely the vector (s^i, σ^i, v^i) subject to the (strong) domain restrictions.[10]

The aggregate savings function is

$$s(p; t) = s^A(p; t^A) + s^B(p; t^B). \tag{8}$$

Setting $\bar{l} = \bar{l}^A + \bar{l}^B$, we obtain from (4), (5), (6) and (8) that

$$\begin{aligned} s_r &= \frac{1}{r^2}(\sigma^A + \sigma^B) - \frac{1}{r}(v^A s^A + v^B s^B); \\ s_w &= \bar{l} - (\bar{l}^A v^A + \bar{l}^B v^B); \\ s_t &= v^B - v^A. \end{aligned} \tag{9}$$

Similarly, we restrict attention to points (k, l) on a neighbourhood of which

(i) the production function $f(k, l)$ is twice continuously differentiable, linear homogeneous, concave and strictly monotonically increasing:

$$f_k > 0 \quad \text{and} \quad f_l > 0;$$

(ii) $f_{kk} < 0$, and hence $f_{lk} > 0$.

Remark 2. Given a vector $(\bar{f}, \bar{f}_k, \bar{f}_{kk})$ with $\bar{f} > 0, \bar{f}_k > 0, \bar{f}_{kk} < 0$, there exists a twice continuously differentiable, linear homogeneous, concave, and strictly monotonically increasing production function f which, at (k, l), satisfies $f = \bar{f}, f_k = \bar{f}_k, f_l = (1/l)[\bar{f} - k\bar{f}_k]$, $f_{kk} = \bar{f}_{kk}$, and $f_{kl} = -(k/l)\bar{f}_{kk}$. Thus we may specify freely the vector (f, f_k, f_{kk}) subject to the domain restrictions.

The linear homogeneity of the production function prevents the derivation of factor demands as functions of factor prices alone. Since, however, the aggregate supply of labour $\bar{l}$ is inelastic, it can be substituted into the first order, necessary and sufficient marginal productivity conditions to yield, first, a well defined demand function for the elastically supplied capital and, subsequently, market clearing prices for the inelastically supplied factors. Consider the capital equation

$$f_k(k, \bar{l}) = r. \tag{10}$$

Since $f_{kk} < 0$, the left-hand side can be inverted to yield a capital demand function $k(r)$. The function $k(r)$ is continuously differentiable, positive, and strictly monotonically decreasing:

$$k' = f_{kk}^{-1} < 0. \tag{11}$$

Next, consider the labour market. If, in addition to $\bar{l}$, we substitute for capital the function $k(r)$, we obtain the wage $w(r)$ necessary and sufficient for market clearing:

$$f_l(k(r), \bar{l}) = w(r). \tag{12}$$

The function w is continuously differentiable, positive, and strictly, differentiably, monotonically decreasing:

$$w' = f_{lk}k' < 0. \tag{13}$$

For a given transfer t, a stationary equilibrium price vector $p(t)$ is such that factor markets clear; equivalently

$$s(r, w(r); t) = k(r)^{11}. \tag{14}$$

Totally differentiating (14) we obtain

$$(1 - s_r f_{kk} - s_w f_{lk})dr = (s_t f_{kk})dt. \tag{15}$$

We restrict our attention to stationary equilibria on a neighbourhood of which

$$1 - s_r f_{kk} - s_w f_{lk} \neq 0. \tag{16}$$

For a fixed level of the transfer t, local dynamic stability of a stationary equilibrium reduces, from (14), to local dynamic stability of the discrete dynamical system $k_{\tau+1} = s(k_\tau)$, where $s(k_\tau) = s(f_k(k_\tau, \bar{l}), f_l(k_\tau, \bar{l}); t)$. It follows that a stationary equilibrium is locally dynamically stable if and only if

$$|s_r f_{kk} + s_w f_{lk}| < 1. \tag{17}$$

Note that local dynamic stability is a stronger condition than local Walrasian stability, the stability notion used in the existing, static literature. From (15), local Walrasian stability reduces to local stability with respect to the dynamical system $\dot{r} = (k(r) - s(r; t))$, which, since $f_{kk} < 0$, is equivalent to

$$1 - s_r f_{kk} - s_w f_{lk} > 0. \tag{18}$$

Let $(p(t), t)$ be a stationary equilibrium. Substituting (9) into (18) and letting

$$H = kf_{kk}\left\{\frac{1}{kf_k^2}[\sigma^A + \sigma^B] - 1 + [v^B - v^A]\frac{\bar{l}^A}{k}\left[\frac{k^A}{\bar{l}^A} - \frac{k}{\bar{l}}\right] + v^B\left[1 - \frac{1}{f_k}\right]\right\} \tag{19}$$

we obtain that the equilibrium is locally dynamically stable if and only if

$$|H| < 1. \tag{20}$$

Substituting (9) into (17) and simplifying, we obtain that

$$\frac{dr}{dt} = \frac{f_{kk}[v^B - v^A]}{1 - H}. \tag{21}$$

Substituting into (7) and simplifying, we obtain that

$$\begin{aligned} \frac{d\tilde{u}^A}{dt} &= \frac{1}{1-H}\left\{[v^B - v^A]\left[\frac{f_{kk}k}{f_k}\frac{\bar{l}^A}{\bar{l}}\right]\left[\frac{k^A/\bar{l}^A}{k/\bar{l}} - f_k\right]\right\} + 1; \\ \frac{d\tilde{u}^B}{dt} &= \frac{1}{1-H}\left\{[v^B - v^A]\left[\frac{f_{kk}k}{f_k}\frac{\bar{l}^B}{\bar{l}}\right]\left[\frac{k^B/\bar{l}^B}{k/\bar{l}} - f_k\right]\right\} - 1. \end{aligned} \tag{22}$$

Equations (21) and (22) express, in terms of the characteristics of the individual preferences, the individual endowments, and the production function, the change in prices and distribution of utility gains and losses resulting from an infinitesimal transfer of income.

Observe that $(1-v^i)$ is the marginal propensity to save in country i and therefore (v^B-v^A) reflects the difference in the marginal propensity to save across countries.

The possibility of the transfer paradox or of a Pareto improving transfer arises when the marginal propensity to save differs across countries.

Local dynamic stability implies that $(1-H)>0$ and is thus related to the distribution of utility gains and losses; it does not, however, unambiguously determine the sign of these changes in utility.

The first term in each of the two equations in (22) is the relative price effect. Taking into consideration (21) we see that the relative price effect is positive for country i if, as a result of the transfer, the equilibrium rate of interest f_k approaches $(k^i/\bar{l}^i)/(k/\bar{l})$. Roughly speaking, the "optimal" rate of interest for country i is $(k^i/\bar{l}^i)/(k/\bar{l})$. The "golden rule" (i.e. $r=1$) maximizes *per capita* income and thus, in a two-country world need not be optimal from any one country's point of view. It is the divergence of optimal rates of interest across countries which, as we shall show in the next section, may allow for the transfer paradox or the phenomenon of a Pareto improving transfer.

3. THE TRANSFER PARADOX AND THE PHENOMENON OF A PARETO IMPROVING TRANSFER

The transfer paradox obtains at a stationary equilibrium if and only if $(d\tilde{u}^A/dt)<0$ while $(d\tilde{u}^B/dt)>0$; the phenomenon of a Pareto improving transfer obtains if and only if $(d\tilde{u}^A/dt)>0$ and $(d\tilde{u}^B/dt)>0$.

To characterize the possibility of occurrence of the transfer paradox or of the phenomenon of Pareto improving transfers we may choose freely the preference and endowment characteristics (k^i, σ^i, v^i), $i=A, B$, subject to the (strong) domain restrictions and the production characteristics (f, f_k, f_{kk}), again subject to the domain restrictions; Remarks 1 and 2 gave the required justification.

Lemma 1. *Necessary conditions for the transfer paradox to occur at locally dynamically stable stationary equilibrium are that*

$$v^B>v^A \quad \textit{and} \quad \frac{k^A/\bar{l}^A}{k/\bar{l}}>f_k>\frac{k^B/\bar{l}^B}{k/\bar{l}},$$

or

$$v^A<v^B \quad \textit{and} \quad \frac{k^B/\bar{l}^B}{k/\bar{l}}>f_k>\frac{k^A/\bar{l}^A}{k/\bar{l}};$$

necessary conditions for the phenomenon of a Pareto improving transfer to occur at locally dynamically stable stationary equilibrium are that

$$v^B>v^A \quad \textit{and} \quad f_k>\frac{k^B/\bar{l}^B}{k/\bar{l}},$$

or

$$v^A>v^B \quad \textit{and} \quad \frac{k^B/\bar{l}^B}{k/\bar{l}}>f_k.$$

Proof. The result follows immediately from (22). ||

We shall now show that the transfer paradox as well as the phenomenon of a Pareto improving transfer may indeed occur.

Note that if at a stationary equilibrium $f_k = 1$ the economy is the golden rule. (We have assumed full depreciation and no population growth). If $f_k > 1 (f_k < 1)$ the economy displays under- (over-) investment.

Substituting (19) into (22) and rearranging terms we obtain

$$\frac{d\tilde{u}^A}{dt} = \frac{1 - kf_{kk}\left\{\frac{1}{kf_k^2}[\sigma^A + \sigma^B] + v^B\left[1 - \frac{1}{f_k}\right] - 1\right\}}{1 - kf_{kk}\left\{\frac{1}{kf_k^2}[\sigma^A + \sigma^B] - 1 + [v^B - v^A]\frac{1}{f_k}\frac{\bar{l}^A}{\bar{l}}\left[\frac{k^A/\bar{l}^A}{k/\bar{l}} - f_k\right] + v^B\left[1 - \frac{1}{f_k}\right]\right\}};$$
$$\frac{d\tilde{u}^B}{dt} = \frac{-1 + kf_{kk}\left\{\frac{1}{kf_k^2}[\sigma^A + \sigma^B] + v^A\left[1 - \frac{1}{f_k}\right] - 1\right\}}{1 - kf_{kk}\left\{\frac{1}{kf_k^2}[\sigma^A + \sigma^B] - 1 + [v^B - v^A]\frac{1}{f_k}\frac{\bar{l}^A}{\bar{l}}\left[\frac{k^A/\bar{l}^A}{k/\bar{l}} - f_k\right] + v^B\left[1 - \frac{1}{f_k}\right]\right\}}. \quad (23)$$

Proposition 1. *The transfer paradox may occur at a locally dynamically stable stationary equilibrium at or away from the golden rule.*

Proof. Suppose that the economy is at the golden rule (i.e. $f_k = 1$). Let $\sigma^i = 0$, $i = A, B$. Then (23) reduces to

$$\frac{d\tilde{u}^A}{dt} = \frac{1 + kf_{kk}}{1 + kf_{kk} - kf_{kk}(v^B - v^A)\left(\frac{k^A}{k} - \frac{\bar{l}^A}{\bar{l}}\right)};$$
$$\frac{d\tilde{u}^B}{dt} = -\frac{1 + kf_{kk}}{1 + kf_{kk} - kf_{kk}(v^B - v^A)\left(\frac{k^A}{k} - \frac{\bar{l}^A}{\bar{l}}\right)}. \quad (24)$$

Since $d\tilde{u}^A/dt = -d\tilde{u}^B/dt$, the transfer paradox occurs if and only if $d\tilde{u}^A/dt < 0$. As follows from (19) the paradox occurs at a stable equilibrium if the denominator of the expressions in (24) is positive. Thus, noting Remarks 1 and 2 we can set $1 + kf_{kk} = -\varepsilon$, where $\varepsilon > 0$, and, for instance, $(v^B - v^A) = 1 - \varepsilon$ and $[k^A/k - \bar{l}^A/\bar{l}] = 2\varepsilon$. Then $d\tilde{u}^A/dt = -\varepsilon/(\varepsilon - 2\varepsilon^3)$, which is negative for ε sufficiently small. By continuity, the argument is immediately extendable to stationary equilibrium away from the golden rule or with $\sigma^i > 0$, $i = A, B$. ||

Proposition 2. *The phenomenon of a Pareto improving transfer may occur at a locally dynamically stable stationary equilibrium, away from the golden rule.*

Proof. Let $\sigma^A = \sigma^B = 0$ and $v^B = 1$ and $v^A = 0$. Then (23) is reduced to

$$\frac{d\tilde{u}^A}{dt} = \frac{1 + \frac{kf_{kk}}{f_k}}{1 + \frac{kf_{kk}}{f_k}\left[1 - \left(\frac{k^A}{k} - \frac{\bar{l}^A}{\bar{l}}f_k\right)\right]};$$
$$\frac{d\tilde{u}^B}{dt} = \frac{-[1 + kf_{kk}]}{1 + \frac{kf_{kk}}{f_k}\left[1 - \left(\frac{k^A}{k} - \frac{\bar{l}^A}{\bar{l}}f_k\right)\right]}. \quad (25)$$

The transfer is Pareto improving if and only if $d\tilde{u}^A/dt > 0$ and $d\tilde{u}^B/dt > 0$. As follows from (19), stability requires that the denominator of the expressions in (25) will be positive. Thus, noting Remarks 1 and 2, we can set $1+kf_{kk} = -\varepsilon$, where $\varepsilon > 0$, and $f_k > 1+\varepsilon$, say $1+2\varepsilon$; then if for instance $\bar{l}^A = \bar{l}^B = 1$ and $k^A/k \leqq (1+\varepsilon)/2$ (say equal), $d\tilde{u}^A/dt = 1$ and $d\tilde{u}^B/dt = \varepsilon$. Note that from (25) it follows immediately that Pareto improvement cannot occur at the golden rule. ||

4. INTERPRETATIONS

If the marginal propensity to save is equal across countries at equilibrium, a transfer of income has no effect on the pattern of investment and therefore on the stationary level of capital in the world economy. Hence, relative prices (terms of trade), which are a function of the level of capital, are not affected by the transfer. The only change in the world economy is the one in the distribution of income which by definition favours the recipient country.[12]

If the marginal propensity to save in the recipient country is lower (higher) than that in the donor country, the transfer decreases (increases) investment, and therefore the steady-state level of capital in the world economy. If competitive equilibrium in this economy is characterized by over (under) investment relative to the golden rule (i.e. $f_k \lesseqgtr 1$), the transfer which decreases (increases) the steady-state level of capital enables the economy to approach the golden rule creating the possibility for Pareto improvement.

If the stationary equilibrium is characterized by the golden rule rate of investment ($f_k = 1$), and if the marginal propensity to save is different across countries, a transfer of income will move the world economy to a state of under- or over-investment. Although the new equilibrium level of investment will then be suboptimal, the resulting equilibrium rate of interest may be closer to the optimal rate of interest from the donor's viewpoint and thus further from the optimal rate of interest from the recipient's viewpoint. The transfer paradox may then occur. (Note that the golden rule rate of investment maximizes per-capita income and thus, in a two-country world, need not be optimal from any one country's viewpoint.) A similar line of argument allows for the transfer paradox away from the golden rule as well.

5. CONCLUSION

The earlier theoretical presumption against the compatibility of the transfer paradox with stability in a world economy with two countries does not extend to a framework of overlapping generations with production and investment.

Away from the golden rule, transfers, possibly motivated by ethical or political considerations, can enhance economic efficiency.

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NOTES

1. A formal treatment was subsequently given by Mundell (1960, 1968). Balasko (1978) refined the argument in the framework of regular economies with two individuals and two goods and drew the distinction between the local and the global version of the paradox for economies with multiple equilibria. The global version involves, of course, welfare comparisons across equilibria. Recently, Bhagwati, Brecher and Hatta

(1983b) argued that, if the Walrasian tâtonnement is relied upon as a global adjustment process, the global paradox with two countries fails even at unstable equilibria.

2. See, for example, Samuelson (1952, 1954), Johnson (1955), and Jones (1970).

3. A link between these two literatures is established in Bhagwati, Brecher, and Hatta (1982).

4. Clearly related are the phenomenon of immiserizing growth as noted by Bhagwati (1958) and the example constructed by Aumann and Peleg (1974) in which an individual can gain by discarding part of the initial endowment.

5. Further elaboration of Gale's example was carried out in the mathematical economics literature by Guesnerie and Laffont (1978) and Safra (1983) and in the trade-theoretic literature by Yano (1983).

6. Johnson (1960), in analyzing the interaction between trade policy and income distribution, discussed the possibility of paradoxical welfare redistribution. An algebraic treatment was subsequently provided by Komia and Shiguki (1967). These contributions can be viewed as treatments of the three-agent transfer problem.

7. The phenomenon was further elaborated upon in Brecher, Bhagwati, and Hatta (1983), Dixit (1983), Geanakoplos and Heal (1983), Jones (1985) and Polemarchakis (1983).

8. Strictly speaking, of course, the number of distinct individual agents in this world is countably infinite. The recursive structure of the economies, however, allows us to consider it as the analogue of a finite world with only two individuals.

9. See Phelps (1961, 1965).

10. For a formal argument, see Geanakoplos and Polemarchakis (1980).

11. Additional, boundary restrictions are required for the existence of a Walrasian equilibrium. For well behaved utility and production functions, the following is a sufficient condition for the existence of a strictly positive equilibrium price vector: Let $g(r)=f_k(s(r), \bar{l}, \bar{x})$; then (i) $\lim_{r\to 0} g(r)=0$ and $\lim_{r\to 0} g'(r)>1$, or $\lim_{r\to 0} g(r)>0$, while (ii) $\lim_{r\to\infty} g(r)=\infty$ and $\lim_{r\to\infty} g'(r)<1$, or $\lim_{r\to\infty} g(\infty)<\infty$. These properties have exact, but cumbersome, analogues in terms of preference and production characteristics. Regularity of the equilibrium is then generic.

12. Equality of the marginal propensity to save across countries obtains, for example, if the world economy aggregates; i.e. when individuals across countries have identical, homothetic preferences. The latter is only a sufficient condition; utility functions of the forms $u^1(c_1, c_2) = c_1^1 + v^1(c_2)$ would yield the result, but not necessarily aggregation.

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