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ON THE TRANSFER PARADOX*

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1. INTRODUCTION

Variations in the distribution of initial endowments induce variations in the Walrasian equilibrium allocation(s) and, hence, in the level of utility attained by the various agents. A reallocation on initial endowments benefits some agents and hurts others. It was Leontief's [1936] original observation that the distribution of utility gains and losses may be perverse: A transfer of resources from one agent to another may lead to a Walrasian equilibrium at which the donor attains a higher and the recipient a lower level of utility compared to the equilibrium associated with the distribution of endowments prior to the transfer. Samuelson [1947, 1952] subsequently argued that this *transfer paradox* is related to the existence of multiple and unstable equilibria. Recently, Balasko [1978] took up Samuelson's argument and refined it within the framework of regular economies. He drew the distinction between the local and global versions of the paradox and demonstrated that, in a regular and smooth exchange economy with two agents and two commodities, (i) for the local transfer paradox to occur it is necessary and sufficient that the Walrasian equilibrium considered be locally unstable, while (ii) for the global transfer paradox to occur at a locally stable Walrasian equilibrium it is necessary and sufficient that there be multiple (and hence at least three) Walrasian equilibria. The association of the transfer paradox with the instability and multiplicity of the Walrasian equilibria for economies with larger numbers of agents and goods was left as an open question. Furthermore, I would argue that the paradox is of interest only either in its local version in the context of a regular economy, or in the context of an economy in which the Walrasian equilibrium is unique; otherwise, it gets confounded with welfare comparisons across equilibria. Thus, the extension to a three or more agent (or good) economy is necessary for the bona fide transfer paradox to be compatible with the stability of the Walrasian tâtonnement.

It is the purpose of this paper to demonstrate that, as long as the number of agents exceeds or is equal to three, the transfer paradox (local or global) can indeed occur even at a unique and stable Walrasian equilibrium. The example consists of a pure exchange economy with two goods and three agents, all of whom

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possess fixed coefficient utility functions. The economy is regular as in Balasko, but of course not smooth; I do not see, however, that the failure of smoothness is essential to the argument. The result is further strengthened by demonstrating that, given any two fixed coefficient agents who are sufficiently diverse so as to allow for the transfer paradox, there exists a regular, fixed coefficient exchange economy, which indeed displays the local transfer paradox between the exogenously specified agents at a locally stable Walrasian equilibrium. Thus, the dismissal of the transfer paradox as empirically irrelevant on grounds of instability is not possible.

A phenomenon similar in spirit to the paradox of transfer is the *phenomenon of advantageous (disadvantageous) reallocation*. A group of agents, by reallocating their initial endowments, may all attain a higher (lower) level of utility at the new equilibrium. Such an example was given by Gale [1974] and the phenomenon was further elaborated upon by Guesnerie and Laffont [1978], and Safra [1981].

To a large extent, the interest of the paradox of transfer as well as of the phenomenon of advantageous or disadvantageous reallocation derives from the fact that the aggregate endowment of the economy remains unaffected. It is less surprising that an agent can benefit by destroying part of his initial endowment. Aumann and Peleg [1971] demonstrated formally this possibility in a two-agent, two-commodity, smooth exchange economy, even under the additional and extreme regularity assumption that all agents have identical homothetic preferences. Bhagwati [1958] had noticed the same phenomenon in the context of growth theory.

Recently, Chichilnisky [1980] has considered the transfer paradox in the context of a fixed coefficient economy. In the main body of the paper, however, she aggregates over utility levels across distinct agents (in particular over the two distinct agents or groups in the "North") some of whom are not involved in the transfer even though they may be affected by it, which requires clarification. It is not evident that the phenomenon she illustrates is the transfer paradox and not the possibility of advantageous (or disadvantageous) reallocation: the utility levels of the recipient and the donor should move in opposite directions for the former but in the same direction for the latter. Furthermore, she does not consider the issue of uniqueness which, as Balasko [1978] has pointed out, is essential. As was pointed out to me, however, by Chichilnisky and Heal, in the last section of the paper, Chichilnisky does indeed consider the conditions under which the utility levels of the donor and the recipient move in the same or in opposite directions. Combined with her earlier results, these conditions can be used to obtain a result very close to the first proposition of this paper. A misunderstanding of Chichilnisky's result contained in Bhagwati and Srinivasan [1982] has been clarified in Geanakoplos and Heal [1982] and, I presume, Chichilnisky [1982]; the latter, however, I have not seen. Bhagwati, Brecher and Hatta [1982], Dixit [1982], and Yano [1981] have extended the analysis to, among others, economies with non-vanishing income effects.

Before proceeding with the formal model, I would like to point out the very simple intuition behind the transfer paradox, which at the same time indicates the appropriateness of the fixed coefficient framework. For the paradox to occur, the change in relative prices following the transfer must be not only favorable to the donor but also strong enough to outweigh the loss in resources on his part. As was very clearly perceived by Samuelson [1952], "...the implied qualitative effect on the terms of trade does not depend upon the price elasticity of one offer curve alone, or on the price elasticities of the two curves; rather the more crucially important parameters are the income elasticities or propensities in the two countries." The fixed coefficient framework eliminates substitution effects and can serve to illustrate the transfer paradox at its starkest.

2. THE ECONOMY

Consider an exchange economy in which agents are indexed by a subscript i, $i=1, 2,\dots, n$. For simplicity only, we support that there are just two commodities indexed by j, $j=a$, b, whose quantities are denoted interchangeably by a subscript j, or simply by a and b. An agent i is characterized by his quasi-concave, monotone utility function u_i defined on his consumption set which is assumed to coincide with the non-negative orthant and his initial endowment vector e_i, a nonzero point in the consumption set. An economy E is a finite collection of agents $E=\{(u_i, e_i), i=1,\dots, n\}=(u, e)$, where $u=(u_1,\dots, u_n)$ defines the distribution of utilities and $e=(e_1,\dots, e_n)$ the distribution of initial endowments. A consumption allocation is a nonnegative vector $x=(x_1,\dots, x_n)$, and a price system a positive vector $p=(p_a, p_b)$. Faced with prices p, agent i expresses excess demand $z_i(p)$ so as to maximize his utility function $u_i(e_i+z)$ subject to the budget constraint $pz\leq 0$; the excess demand may of course be a correspondence. Aggregate excess demand is denoted by $z(p)=\sum_{i=1}^{n} z_i(p)$. A Walrasian equilibrium for the economy $E=(u, e)$ is a pair $w=(\bar{p}, \bar{x})$ of a price system $\bar{p}$ and a consumption allocation $\bar{x}$, such that $(\bar{x}_i-e_i)\in z_i(\bar{p})$ for all i, $\sum_{i=1}^{n}(\bar{x}_{ij}-e_{ij})\leq 0$, and $\sum_{i=1}^{n} p_j(\bar{x}_{ij}-e_{ij})=0$ for all j. An economy may of course have multiple Walrasian equilibria; the set of Walrasian equilibria of the economy E is denoted by $W(E)$. The utility level attained by agent $i\in E$ at the Walrasian equilibrium $w\in W(E)$ is denoted by $u_i(w)$.

An economy E is said to be *regular*[1] if and only if the following conditions are satisfied: (1) For each agent i the initial endowment e_i is strictly positive, and the excess demand function a well-defined and continuously differentiable function on the strictly positive price-endowment domain; (ii) For each commodity j, for price systems sufficiently close to the boundary $\partial_j=\{p=(p_a, p_b)>0 \mid p_j=0,$

[1] The definition of regularity differs from the standard definition due to Debreu [1970] in that as the price of a commodity tends to zero the corresponding element of the excess demand vector is required to be positive but not necessarily unboundedly so. This is natural in the present context since we do not consider possibly unbounded variations in the initial endowments.

$p_{j'} \neq 0, j \neq j'\}$, the aggregate excess demand for commodity j, $z_j(p)$, is positive, $z_j(p) > 0$; (iii) The jacobian of the aggregate excess demand function $|D(p)|$ never vanishes at an equilibrium: $z(\bar{p}) = 0 \Rightarrow |D(\bar{p})| \neq 0$.

The stability of a Walrasian equilibrium $w = (\bar{p}, \bar{x}) \in W(E)$ refers to the stability of the Walrasian tatonnement defined by $\dot{p} = z(p)$. In the case of a regular economy with two commodities, the criterion for stability is particularly simple: A Walrasian equilibrium is locally (asymptotically) stable if and only if $\frac{\partial}{\partial p_j} z_j(\bar{p}) < 0$, $j = a, b$. The following properties of regular economies with two commodities follow from elementary differential topology:

(i) A regular economy has a finite (odd) number of locally unique Walrasian equilibria, all associated with strictly positive prices;

(ii) Along the normalized strictly positive price domain $P = \{p \gg 0 \mid p_b = 1\}$, the Walrasian equilibria alternate between (odd) locally stable and (even) locally unstable. If the Walrasian equilibrium is (globally) unique, it is (globally asymptotically) stable: $(p_a - \bar{p}_a) z_a(p) < 0$ at all $p \neq \bar{p}$;

(iii) Holding the distribution of utilities $u = (u_1, \ldots, u_n)$ fixed, we may identify an economy $E = (u, e)$ with the distribution of initial endowments $e = (e_1, \ldots, e_n)$, $E \simeq (e)$. If $e \in \dot{R}^{2n}_+$ is a regular economy and $w \in W(e)$, there is a neighborhood V of e in $\dot{R}^{2n}_+$ and a continuously differentiable function $g_w \colon V \to P \times \dot{R}^{2n}_+$, such that $g_w(e) = w \in W(e)$, $g_w(e') = w' \in W(e')$ and $W(e') = \bigcup_{w \in W(e)} g_w(e)$ for all $e' \in V$; we call w and w' *associated equilibria*.

We can now describe precisely the circumstances under which the transfer paradox and the phenomenon of advantageous reallocation are said to occur:

An economy $E = (u, e)$ displays the *local transfer paradox* if and only if there exist two distinct groups of agents $I_1, I_2 \subset \{1, \ldots, n\}$, $I_1 \cap I_2 = \emptyset$, a distribution of initial endowments e' arbitrarily close to e, and associated Walrasian equilibria $w \in W(e)$ and $w' \in W(e')$ such that

a) $e_i = e'_i$ for all $i \notin I_1 \cup I_2$;

b) $e_{i_1} > e'_{i_2}$ for all $i_1 \in I_1$, $e_{i_2} < e'_{i_2}$ for all $i_2 \in I_2$, and $\sum_{I_1 \cup I_2} e_i = \sum_{I_1 \cup I_2} e'_i$;

c) $u_{i_1}(w) < u_{i_1}(w')$ for all $i_1 \in I_1$, and $u_{i_2}(w) > u_{i_2}(w')$ for all $i_2 \in I_2$.

More precisely, we may say that the economy E displays the local transfer paradox between agents in I_1 and I_2 at w. Agents I_1 are referred to as *donors* and agents I_2 as *recipients*. If e' and w' exist but not necessarily arbitrarily close to e and w, respectively, we speak of the *global transfer paradox*; it is clearly weaker than the local transfer paradox

An economy $E = (u, e)$ allows for a *local advantageous reallocation of initial endowments* if and only if there exist a subset of agents $I \subset \{1, \ldots, n\}$, a distribution of initial endowments e' arbitrarily close to e, and associated Walrasian equilibria $w \in W(e)$ and $w' \in W(e')$ such that

a) $e_i = e'_i$ for all $i \notin I$;

b) $\sum_{i \in I} e_i = \sum_{i \in I} e'_i$;

c) $u_i(w) < u_i(w')$ for all $i \in I$.

More precisely we may say that the economy E allows for a local advantageous reallocation of initial endowments among agents in I at w. If e' and w' exist but not necessarily arbitrarily close to e and w, respectively, we speak of the *global advantageous reallocation of initial endowments*; it is clearly weaker than the local. A disadvantageous reallocation is defined analogously.

Concerning these definitions a few remarks are in order:

(i) The possibility of multiple Walrasian equilibria introduces an element of ambiguity in the definition of the circumstances under which the global version of the phenomenon can be said to occur: The existential quantifier allows for comparisons across equilibria, which is hardly meaningful. At the local level, of course, the ambiguity is eliminated by the local uniqueness of the Walrasian equilibria implied by regularity.

(ii) The transfer paradox was originally considered in the framework of a two-agent economy in which, following a transfer (or any reallocation for that matter), the utilities of the two agents are bound to move (if at all) in opposite directions. I have decided to introduce this as a requirement in the definition of the paradox even in the many-agent case and to refer to all cases in which utilities move in the same direction as advantageous (or disadvantageous) reallocations.

The framework within which we shall consider the transfer paradox (and the possibility of advantageous reallocation) is that of a *regular*, *fixed coefficient economy*. An agent (w_i, e_i) is said to be a *regular*, *fixed coefficient agent* if and only if his initial endowment vector $e_i=(\bar{a}_i, \bar{b}_i)$ is strictly positive and his utility function is of the form $u_i(a, b)=\min(\lambda_i a, b)$, $\lambda_i>0$; thus, he is unambiguously defined by the triple $(\lambda_i, \bar{a}_i, \bar{b}_i)$. The excess demand function of the agent $(\lambda_i, \bar{a}_i, \bar{b}_i)$ is given by

$$(1)\quad \begin{aligned} &\text{(a)} \quad z_{ia}(p_a, p_b)=\frac{p_b(\bar{b}_i-\lambda_i\bar{a}_i)}{p_a+\lambda_i p_b}\,; \\ &\text{(b)} \quad z_{ib}(p_a, p_b)=\frac{p_a(\bar{a}_i-(1/\lambda_i)\bar{b}_i)}{(1/\lambda_i)p_a+p_b}\,. \end{aligned}$$

The aggregate excess demand function of an economy of regular, fixed coefficient agents is thus given by

$$(2)\quad \begin{aligned} &\text{(a)} \quad z_a(p_a, p_b)=\sum_{i=1}^{n}\frac{p_b(\bar{b}_i-\lambda_i\bar{a}_i)}{p_a+\lambda_i p_b}\,; \\ &\text{(b)} \quad z_b(p_a, p_b)=\sum_{i=1}^{n}\frac{p_a(\bar{a}_i-(1/\lambda_i)\bar{b}_i)}{(1/\lambda_i)p_a+p_b}\,. \end{aligned}$$

It follows that an economy $E=\{(\lambda_i, a_i, b_i), i=1,\ldots, n\}$ of regular, fixed coefficient agents is a *regular, fixed coefficient economy* if and only if

$$(3)\quad \begin{aligned} &\text{(a)}\quad \sum_{i=1}^{n} \frac{\bar{b}_i - \lambda_i \bar{a}_i}{\lambda_i} > 0; \\ &\text{(b)}\quad \sum_{i=1}^{n} \frac{\bar{a}_i - (1/\lambda_i)\bar{b}_i}{(1/\lambda_i)} > 0. \end{aligned}$$

Note that in the case of just one agent, $n=1$, (3.a) and (3.b) are incompatible. This is due to the fact that a fixed coefficient agent is a net supplier or net demander of a commodity irrespective of relative prices. For $n \geq 2$ regularity is, of course, possible. To simplify the exposition, we now normalize prices by setting the price of commodity b, the numeraire, equal to unity: $p=(p_a, p_b)=(p, 1) \gg 0$, and we restrict our attention to the individual and aggregate excess demand for commodity a, which we denote by $z_i(p)$, $i=1,\ldots, n$ and $z(p)$, respectively. Furthermore, we introduce the auxiliary parameters

$$(4)\qquad \gamma_i = \bar{b}_i - \lambda_i \bar{a}_i, \qquad i = 1,\ldots, n,$$

and observe that individual and hence, aggregate excess demand behavior is completely described by the set of parameters $\{(\lambda_i, \gamma_i), i=1,\ldots, n\}$:

$$(5)\qquad z_i(p) = \frac{\gamma_i}{p+\lambda_i}, \qquad i = 1,\ldots, n$$

$$(6)\qquad z(p) = \sum_{i=1}^{n} \frac{\gamma_i}{p+\lambda_i}.$$

Since to each pair (λ_i, γ_i) with $\lambda_i > 0$ and γ_i arbitrary there corresponds a continuum and hence, at least one regular, fixed coefficient agent $(\lambda_i, \bar{a}_i, \bar{b}_i)$, we shall often identify a regular, fixed coefficient agent with the pair (λ_i, γ_i), $\lambda_i > 0$, and a regular, fixed coefficient economy with the set $E=\{(\lambda_i, \gamma_i), i=1,\ldots, n\}$ of regular, fixed coefficient agents, which, in addition, satisfies the conditions

$$(7)\quad \begin{aligned} &\text{(a)}\quad \sum_{i=1}^{n} \frac{\gamma_i}{\lambda_i} > 0; \\ &\text{(b)}\quad \sum_{i=1}^{n} \gamma_i < 0; \end{aligned}$$

they are obtained from (3) by substitution from the definition of γ_i, $i=1,\ldots, n$ in (4). Observe that the agent (λ_i, γ_i) has net excess supply (demand) for commodity a if and only if $\gamma_i < 0$ $(\gamma_i > 0)$, independently of $p > 0$; if $\gamma_i = 0$, the agent's excess demand vanishes independently of p.

A Walrasian equilibrium price system is a price $\bar{p}$ such that

$$(8)\qquad z(\bar{p}) = \sum_{i=1}^{n} \frac{\gamma_i}{\bar{p}+\lambda_i} = 0.$$

Furthermore, $\bar{p}$ is locally stable if and only if $z'(\bar{p}) < 0$ or equivalently

$$(9)\qquad z'(\bar{p}) = -\sum_{i=1}^{n} \frac{\gamma_i}{(\bar{p}+\lambda_i)^2} < 0.$$

The utility level attained by the agent $(\lambda_i, \bar{a}_i, \bar{b}_i)$ at $\bar{p}$ is

$$\bar{u}_i = \frac{\bar{p}\bar{a}_i + \bar{b}_i}{(1/\lambda_i)\bar{p}+1}, \qquad i = 1,\ldots, n. \tag{10}$$

The equilibrium price level $\bar{p}$ (and hence the utility level $\bar{u}_i$, $i=1,\ldots, n$) is a continuously differentiable function of the endowment parameters $(\ldots, a_i,\ldots, b_i,\ldots)$ or $(\ldots\gamma_i,\ldots)$. Differentiating (8) implicitly and rearranging we obtain

$$\frac{d\bar{p}}{d\gamma_k} = \frac{1}{\bar{p}+\lambda_k}\left[\sum_{i=1}^{n} \frac{\gamma_i}{(\bar{p}+\lambda_i)^2}\right]^{-1}, \qquad k = 1,\ldots, n. \tag{11}$$

3. THE TRANSFER PARADOX

Consider now a transfer $\tau \geq 0$ in terms of the numeraire commodity b from agent $i_1=1$ to agent $i_2=2$. The economy $E(\tau)$ differs from the economy $E \equiv E(0)$ only in that $\bar{b}_1(\tau)=\bar{b}_1-\tau$ while $\bar{b}_2(\tau)=\bar{b}_2+\tau$; all other preference and endowment characteristics are independent of τ. By regularity, for τ small at least, the price level $\bar{p}$ (and hence the utility level $\bar{u}_i$, $i=1,\ldots, n$) is a continuously differentiable function of τ. From (10) it follows that, up to a positive multiple,

(12)

(a) $$\left.\frac{d\bar{u}_1}{d\tau}\right|_{\tau=0} = -1 - (1/\lambda_1)\bar{p} - (1/\lambda_1)\gamma_1 \left.\frac{d\bar{p}}{d\tau}\right|_{\tau=0}$$

(b) $$\left.\frac{d\bar{u}_2}{d\tau}\right|_{\tau=0} = 1 + (1/\lambda_2)\bar{p} - (1/\lambda_2)\gamma_2 \left.\frac{d\bar{p}}{d\tau}\right|_{\tau=0}$$

(c) $$\left.\frac{d\bar{u}_i}{d\tau}\right|_{\tau=0} = -(1/\lambda_i)\gamma_i \left.\frac{d\bar{p}}{d\tau}\right|_{\tau=0}, \qquad i = 3,\ldots, n.$$

The first two terms in (12.a) and (12.b) capture the change in utility due to the transfer in the absence of a price change; this *pure transfer effect* is unambiguously negative for the donor and positive for the recipient. The last terms in (12.a) and (12.b) and the only term in (12.c) are due to the change in relative prices (terms of trade) induced by the transfer. Observe that, as pointed out earlier, for $\gamma_i<0$ agent i supplies commodity a and benefits from an increase in its price — $(d\bar{p}/d\tau)>0$; the contrary is true for $\gamma_i>0$. The transfer may turn out to benefit the donor and hurt the recipient if this *relative price effect* is opposite in sign and large in absolute value relative to the pure transfer effect.

To estimate $(d\bar{p}/d\tau)$ at $\tau=0$ observe that $(d\gamma_1/d\tau)=-1$, $(d\gamma_2/d\tau)=1$, and $(d\gamma_i/d\tau)=0$ for $i=3,\ldots, n$. From (8) it follows that

$$\frac{d\bar{p}}{d\tau} = \left[\frac{1}{\bar{p}+\lambda_2} - \frac{1}{\bar{p}+\lambda_1}\right]\left[\sum_{1=1}^{n} \frac{\gamma_i}{(\bar{p}+\lambda_i)^2}\right]^{-1}. \tag{13}$$

The economy E displays the local paradox of transfer at p if and only if $(d\bar{u}_1/d\tau)>0$ and $(d\bar{u}_2/d\tau)<0$ at $\tau=0$. It follows from (12.a), (12.b), and (13) that the following conditions relating the characteristics of the donor and the recipient are necessary for the paradox to occur:

$$(14)\quad \begin{aligned} &\text{(a)}\quad \gamma_1\gamma_2 < 0; \\ &\text{(b)}\quad (\gamma_2-\gamma_1)(\lambda_2-\lambda_1) < 0; \end{aligned}$$

condition (14.a) is equivalent to the statement that the two agents are at opposite sides of the market; to interpret condition (14.b) observe that the local stability of the equilibrium $\bar{p}$ is equivalent to the second term in the expression for $(d\bar{p}/d\tau)$ in (13) being positive, and hence, (14.b) simply guarantees that the transfer induces an increase of the price of the commodity supplied by the donor. Observe, furthermore, that the conditions (14) necessary for the local transfer paradox to occur between the agents $i=1$ and 2 at a locally stable Walrasian equilibrium are independent of the characteristics of the other agents in the economy. Thus, if (14) holds, we shall say that the agents (λ_1, γ_1) and (λ_2, γ_2) allow for the transfer paradox.

PROPOSITION 1. *There exists a regular, fixed coefficient economy with three agents, which displays the local transfer paradox at a unique and stable Walrasian equilibrium.*

PROOF. The argument is constructive. Let

$$\lambda_1 = 1\tfrac{2}{5} = 1.4 \qquad \gamma_1 = -4\tfrac{2}{5} = -4.4$$

$$\lambda_2 = \tfrac{7}{10} = .7 \qquad \gamma_2 = 2\tfrac{3}{5} = 2.6$$

$$\lambda_3 = 2\tfrac{3}{5} = 2.6 \qquad \gamma_3 = 1\tfrac{27}{34} \approx 1.1$$

and observe that $\sum_{i=1}^{3} \gamma_i \approx -.7$, while $\sum_{i=1}^{3} \frac{\gamma_i}{\lambda_i} \approx .9$; the economy is regular. By construction $\sum_{i=1}^{3} \frac{\gamma_i}{1+\lambda_i} = 0$, and $\bar{p}=1$ is a Walrasian equilibrium. Since $\sum_{i=1}^{3} \gamma_i \neq 0$, the equation $z(p) = \sum_{i=1}^{3} \frac{\gamma_i}{p+\lambda_i}$ has at most two real roots, while regularity implies that the number of positive ones is odd; (global) uniqueness and hence (global) stability of the Walrasian equilibrium price $\bar{p}=1$ follow.

To complete the argument it remains to compute $(d\bar{u}_1/d\tau)$ and $(d\bar{u}_2/d\tau)$ at $\tau=0$. Substituting in (13) we obtain that $(d\bar{p}/d\tau) \approx .847$. Finally, substituting into (12.a) and (12.b) we obtain that, up to a positive multiple, $(d\bar{u}_1/d\tau) \approx 1.34$ while $(d\bar{u}_2/d\tau) \approx -.5$ at $\tau=0$. The paradox follows. Q.E.D.

The example given in the proof of Proposition 1 is minimal: as is well-known regularity prevents the occurrence of the local transfer paradox from occuring at a locally stable Walrasian equilibrium in an economy with two agents. In the fixed coefficient framework in particular, the Walrasian equilibrium allocation with only two agents is independent of small variations in the distribution of initial endowments and therefore, trivially, the paradox fails.

PROPOSITION 2. *Let* $(\lambda_1^*, \gamma_1^*)$ *and* $(\lambda_2^*, \gamma_2^*)$ *be two regular, fixed coefficient agents* *who allow for the transfer paradox. There exists a regular, fixed coefficient economy, which displays the local paradox of transfer between agents* 1 *and* 2 *at a locally stable Walrasian equilibrium.*

PROOF. We shall construct a regular, fixed coefficient economy with six agents, $E=\{(\lambda_i, \gamma_i), i=1,\dots, n\}$, with $(\lambda_i, \gamma_i)=(\lambda_i^*, \gamma_i^*)$, $i=1, 2$, which displays the transfer paradox between the agents $i_1=1$ and $i_2=2$ at a locally stable Walrasian equilibrium $\bar{p}$. Since the agents $(\lambda_1^*, \gamma_1^*)$ and $(\lambda_2^*, \gamma_2^*)$ allow for the paradox, we may without loss of generality assume that $\gamma_1<0$, $\gamma_2>0$ and $\lambda_1>\lambda_2$. From (11) it follows that there exists $M>0$ such that the $(\partial\bar{u}_1/d\tau)>0$ and $(\partial\bar{u}_2/d\tau)<0$ at $\tau=0$ provided $(d\bar{p}/d\tau)\geq M$. Consider then the following system:

$$(15)\qquad \begin{aligned} &\text{(a)} \quad \sum_{i=1}^{6} \gamma_i < 0; \\ &\text{(b)} \quad \sum_{i=1}^{6} \frac{\gamma_i}{\lambda_i} > 0; \\ &\text{(c)} \quad \sum_{i=1}^{6} \frac{\gamma_i}{1+\lambda_i} = 1; \\ &\text{(d)} \quad \sum_{i=1}^{6} \frac{\gamma_i}{(1+\lambda_i)^2} \geq M. \end{aligned}$$

(15.a) and (15.b) guarantee regularity (from (7)); (15.c) implies (from (8)) that $\bar{p}=1$ is a Walrasian equilibrium price system; (15.d) implies simultaneously that $\bar{p}=1$ is locally stable (from (9)) and that the paradox obtains (from (11)). To complete the argument it suffices to show that there exist $\{(\gamma_i, \lambda_i), i=3,\dots, 6\}$ such that the economy E satisfies (15). Observe now that, for fixed $(\lambda_1^*, \gamma_1^*)$ and $(\lambda_2^*, \gamma_2^*)$ as well as $(\lambda_3, \lambda_4, \lambda_5, \lambda_6)$, the system is linear in $(\gamma_3, \gamma_4, \gamma_5, \gamma_6)$. To obtain a solution then it suffices to set λ_i, $i=3,\dots, 6$ arbitrarily provided only that they are chosen positive and such that the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ \frac{1}{\lambda_3} & \frac{1}{\lambda_4} & \frac{1}{\lambda_5} & \frac{1}{\lambda_6} \\ \frac{1}{\lambda_3+1} & \frac{1}{\lambda_4+1} & \frac{1}{\lambda_5+1} & \frac{1}{\lambda_6+1} \\ \frac{1}{(\lambda_3+1)^2} & \frac{1}{(\lambda_4+1)^2} & \frac{1}{(\lambda_5+1)^2} & \frac{1}{(\lambda_6+1)^2} \end{bmatrix}$$

be of full rank, and then solve for γ_i, $i=3,\dots, 6$. That A can indeed be chosen to be of full rank is evident — the determinant is a non-zero polynomial in λ_3, λ_4, λ_5, λ_6. Q.E.D.

The paradox occurs when the price effect due to the transfer is not only advantageous to the donor but also strong enough to overcome the pure transfer effect.

Intuition then suggests that as the economy gets large the paradox should eventually disappear: the price effect tends to zero while the pure transfer effect is independent of the size of the economy. To see that this intuition is indeed correct, let E_m be the m-fold replica of the regular, fixed coefficient economy $E=\{(\lambda_i, \gamma_i), i=1,\ldots, n\}$ — the index i identifies now types as opposed to individual agents. It follows directly from (13) that the price effect $(d\bar{p}/d\tau)$ of a transfer between two individual agents of type $i_1=1$ and $i_2=2$ is given by

$$(16) \qquad \frac{d\bar{p}}{d\tau} = \left[\frac{1}{\bar{p}+\lambda_2} - \frac{1}{\bar{p}+\lambda_1}\right]\left[m\sum_{i=1}^{n} \frac{\gamma_i}{(\bar{p}+\lambda_1)^2}\right]^{-1}.$$

Since by regularity $\sum_{i=1}^{n} \frac{\gamma_i}{(\bar{p}+\lambda_1)^2} = -z'(p)\neq 0$, $(d\bar{p}/d\tau)$ can be made arbitrarily small by an appropriate choice of size, m. It then follows from (12.a) and (12.b) that for $m\geq \bar{m}$, some $\bar{m}>0$, $(\partial u_1/d\tau)<0$ and $(\partial u_2/d\tau)>0$.

The question can be raised whether the welfare consequences of a transfer depend on the good (a or b) that is actually transferred. It is a straightforward computation to show that, independently of the utility functions of the agents, $\frac{du_i}{d\tau_a} = \bar{p}\,\frac{du_i}{d\tau_b}$, where τ_a and τ_b denote transfers in terms of good a and b, respectively. It is, thus, immaterial which good the transfer occurs in; only purchasing power matters.

4. THE PHENOMENON OF ADVANTAGEOUS REALLOCATION

To simplify the analysis, it is convenient to consider first the case of a single agent who attains a higher level of utility by destroying part of his endowment. To illustrate this phenomenon in the present framework, consider the economy $E(\tau)$, $\tau\geq 0$, which differs from the economy $E\equiv E(0)$ only in that $b_1(\tau)=\bar{b}_1-\tau$; all other preference and endowment characteristics are independent of τ. Proceeding as previously, we obtain that, up to a positive multiple,

$$(17) \quad \begin{aligned} &\text{(a)} \quad \left.\frac{d\bar{u}_i}{d\tau}\right|_{\tau=0} = -1-(1/\lambda_1)\bar{p}-(1/\lambda_1)\gamma_1\left.\frac{d\bar{p}}{d\tau}\right|_{\tau=0}; \\ &\text{(b)} \quad \left.\frac{d\bar{u}_i}{d\tau}\right|_{\tau=0} = -(1/\lambda_1)\gamma_i\left.\frac{d\bar{p}}{d\tau}\right|_{\tau=0}, \qquad i=2,\ldots,n. \end{aligned}$$

The reduction in his endowment may indeed benefit the agent even in an economy with two agents and a unique and stable Walrasian equilibrium: Note that for $n=2$

$$(18) \qquad \bar{p} = -\frac{\lambda_1\gamma_2+\gamma_1\lambda_2}{\gamma_1+\gamma_2};$$

since $\gamma_1=\bar{b}_1-\lambda_1\bar{a}_1-\tau$,

$$(19) \qquad \frac{d\bar{p}}{d\tau} = \frac{\gamma_2(\lambda_2-\lambda_1)}{(\gamma_1+\gamma_2)^2};$$

substituting from (19) into (16.a) we obtain that, up to a positive multiple,

$$\frac{du_1}{d\tau}\bigg|_{\tau=0} = \lambda_2 - \lambda_1 \tag{20}$$

Since from the regularity conditions (7), $\lambda_2 > \lambda_1$ if and only if $\gamma_1 > 0$, the interpretation is clear: the agent benefits from the reduction of part of his endowment in a commodity of which he is a net supplier.

From this point on, it is easy to demonstrate the phenomenon of advantageous reallocation. In the economy considered above, adjoin an agent $i=3$ whose preferences and endowments involve only commodity b; i.e. $u_3(a, b) = b$, $\lambda_3 = 0$, and $e_3 = (0, \bar{b}_3)$. Then, transferring commodity b from agent 1 to 3 is tantamount to destroying it, other than that it increases the utility level of agent 3. Finally, setting $\lambda_3 > 0$ and $a_3 > 0$ but arbitrarily close to 0 establishes regularity for the augmented economy, while, by continuity, the phenomenon of advantageous reallocation among agents in $I = \{1, 3\}$ still obtains.[2]

5. CONCLUSION

The extension to more than three commodities is of no particular interest at least under fixed coefficients: the paradox cannot occur in an economy with three or more commodities but just two agents. It is of interest, on the other hand, to consider general preferences and determine the conditions under which the paradox occurs. If the analogues of Propositions 1 and 2 are valid, the paradox of transfer cannot be considered either pathological or unlikely to occur and the intriguing question follows whether aid which is guaranteed to benefit the recipient is possible in a decentralized framework.

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[2] Safra [1981] has established that in economies with unique Walrasian equilibria the phenomenon of advantageous reallocation cannot occur. The results here are not in contradiction to Safra's, since he requires uniqueness to prevail independently of the distribution of initial endowments, allowing even for negative endowments.

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