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Retroactive Money

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INTRODUCTION

This paper makes three main points. First, monetary policy can influence real economic activity even in a regime of rational expectations and flexible prices; active monetary policy is furthermore likely to be desirable. Second, it is possible, in a model in which anticipated and unanticipated monetary disturbances have different effects, for an econometrician mistakenly to conclude that monetary policy is ineffective (e.g. to estimate a vertical Phillips curve) when it is omnipotent. Third, even if monetary policy cannot be quickly implemented (the government needs time to respond to economic variables), policy is still effective; in such a model an increase in the money stock leads to increased short-run output but decreased future output, and effective policy causes a business cycle in which output is serially correlated.

That policy may be desirable is a point well established in economic theory (Diamond, 1965). It has been argued, on the other hand (Lucas, 1972), that, as long as agents form their expectations concerning future variables rationally, any attempt towards active policy (in particular, monetary policy) anticipated by the agents will fail; the allocation of resources will not be affected. The argument is then further developed to demonstrate that unanticipated policy changes can indeed affect real variables as long as agents act under imperfect information concerning the actual state of the economy. The vehicle through which policy changes are effective is the inability of decision-making agents to distinguish between "real" and "nominal" shifts (Lucas, 1972; Polemarchakis and Weiss, 1977; Weiss, 1980).

We shall demonstrate that, even in the context of an economy with homogeneous information and perfect stochastic foresight, monetary policy can be effective. Furthermore, under an expected utility criterion active monetary invervention may improve upon the allocation obtained with a fixed money supply.

I. A SIMPLE MODEL WITH OMNIPOTENT POLICY

Consider an economy[1] extending over discrete time, t. During each time period, two generations are alive—one young and one old. Old people consume but do not work; young people work but do not consume. Agents live for two periods; their endowment is their labor when young, and their intertemporal von Neumann–Morgenstern utility function has the form[2]

$$(1)\qquad u(l,c)=-l+\frac{1}{\alpha}c^{\alpha},\qquad \alpha<1,\alpha\neq 0$$

where l is the labour supplied when young and c is the consumption when old. There is no capital in the economy and no growth in population. Young agents work so as to produce the consumption good according to the production function

(2) $c = \theta l, \qquad \theta > 0$

which they sell to the old in exchange for fiat money. Old agents spend their money holdings so as to purchase the consumption good. The total stock of money is M.

Uncertainty is introduced as follows. There are n types of generations indexed by a subscript $k(i)$, $k = 1, \ldots, n$, $(i = 1, \ldots, n)$ and distinguished by the productivity of their labour—i.e. by the parameter θ of the production function (2). There are no intra-generational differences. The random variable θ is distributed identically and independently over time;[3] it takes on the value θ_k with probability π_k.

The tool at the disposal of the government is monetary policy. It chooses (possibly negative) lump-sum payments, s, to be distributed in the following period to what is the young generation of the current period.[4] The government's decision is assumed to be made before the young generation decide on their labour supply. Furthermore, since it is a lump-sum transfer, the amount of money balances they are going to receive is correctly perceived by young agents to be indepeendent of the amount they themselves decide to carry over. Finally, we assume that the government learns the current value of the random productivity level θ before it has to decide on the level of the lump-sum transfers. Observe that, even though this is a strong assumption it does *not* involve an assumption of informational superiority of the government as compared with the other agents in the economy, or "the market".

A *monetary policy* is a function $s(\theta, M)$ such that $s(\theta, M) + M > 0$, and a *price system* is a pair of functions $[p(\theta, M), w(\theta, M)]$ where p refers to the nominal price of the consumption good and w to the nominal wage rate. Since the variable θ takes on finitely many values, and since the dependence on M is always linear, we often refer to a monetary policy as a vector $\mathbf{s} = (s_1, \ldots, s_k, \ldots, s_n)$ and to a price system as a pair of vectors $[\mathbf{p}, \mathbf{w}] = [(p_1, \ldots, p_k, \ldots, p_n), (w_1, \ldots, w_k, \ldots, w_n)]$ for $M' \neq M$,

$$p(\theta_k, M') = \frac{M'}{M} p_k, \qquad w(\theta_k, M') = \frac{M'}{M} w_k \quad \text{and} \quad S(\theta_k, M) = \frac{M'}{M} s_k.$$

Observe that we define a policy and a price system as functions of the *state of the economy* (θ, M). We do not allow them to depend on the history of the economy or, equivalently, on calendar time. There is no good justification for such a simplifying assumption other than that it is necessary in order to avoid problems of non-uniqueness of equilibrium.

Given a monetary policy, $\mathbf{s}$, and a price system, $[\mathbf{p}, \mathbf{w}]$, a young person of type k decides on how much labour to supply by solving the following maximization problem:

(3) $\max_{l, c_1, \ldots, c_n} \; -l + \frac{1}{\alpha} \sum_i \pi_i c_i^{\alpha}$

$$\text{s.t.}\quad p_i\left(\frac{M+s_k}{M}\right)c_i = w_k l + s_k, \qquad i=1,\ldots,n$$

$$l \geq 0,\ c_i > 0, \qquad i=1,\ldots,n$$

since, if θ takes the value θ_i in the following period, the price of the consumption good will be $p_i\{(M+s_k/M)\}$. The factor multiplying p_i arises out of the degree one homogeneity of prices with respect to the money stock Agents act with full knowledge of the mechanism that generates future prices, as required by the postulate of rational expectations. The labour supply function can be explicitly computed:

$$(4)\qquad l_k(\mathbf{s}, \mathbf{p}, \mathbf{w}) = \frac{1}{w_k}\left\{\left(w_k \sum_i \pi_i p_i^{-\alpha}\right)^{1/(1-\alpha)}\left(\frac{M}{M+s_k}\right)^{\alpha/(1-\alpha)} - s_k\right\}, \qquad k=1,\ldots,n.$$

An old person has a trivial decision problem: he spends his entire money balances to acquire the consumption good. The production sector is indifferent between any level of employment and production as long as

$$(5)\qquad w_k = \theta_k p_k, \qquad k=1,\ldots,n.$$

We can now define an equilibrium for the economy.

Definition 1. A monetary policy $s(\theta, M)$ and a price system $[p(\theta, M), w(\theta, M)]$ are an equilibrium for the economy if and only if:

$$(6a)\qquad w_k l_k = M, \qquad k=1,\ldots,n$$

$$(6b)\qquad w_k = \theta_k p_k, \qquad k=1,\ldots,n.$$

Equation (6a) corresponds to equilibrium in the money market. By the linearity of the technology, equation (6b) guarantees equilibrium in the labour market. The goods market then clears by Walras's law.

Proposition 1. *Given a monetary policy $s(\theta, M)$, there exists a unique price $[p(\theta, M), w(\theta, M)]$ such that the triple $[s(\theta, M, p(\theta, M), w(\theta, M)]$ is an equilibrium for the economy.*

Proof. For

$$b = \left\{\sum_k \pi_k \theta_k^\alpha \left(\frac{M}{M+s_k}\right)^\alpha\right\}^{1/(\alpha-1)}$$

let

$$p_k = (M+s_k)\frac{b}{\theta_k}$$

$$(7)\qquad w_k = (M+s_k)b$$

$$l_k = \frac{M}{(M+s_k)}\frac{1}{b}.$$

The system of equations (6) in Definition 1 is then clearly satisfied Q.E.D.

We then use the notation $\sigma(\theta, M)$ for the ratio $\{s(\theta, M)+M\}/M$.

In the context of this simple economy, all real variables are determined if we specify the labour supply function $l(\theta)$—the labour supplied by the young generation as a function of their productivity. To raise the question of the effectiveness of monetary policy then, one must ask whether different equilibrium labour supply functions can result from different monetary policy functions. The question so posed is well defined in light of Proposition 1. It can be shown that monetary policy is not only effective; it is omnipotent.

Proposition 2. *Let $f(\theta)$ be an arbitrary positive function. There exists a unique monetary policy $s(\theta, M)$ and a unique equilibrium $[s(\theta, M), p(\theta, M), w(\theta, M)]$ such that the equilibrium labour supply function $l(\theta)$ is equal to $f(\theta)$.*

Proof. The function $f(\theta)$ defines uniquely the wage function

$$w(\theta, M)=M/f(\theta),$$

from (6b). The wage function $w(\theta, M)$ then defines uniquely the price function $p(\theta, M)=w(\theta, M)/\theta$, from (6a). Finally, the required monetary policy is defined by

$$M+s_k=M^{\alpha}(\theta_k p_k)\Big(\sum_i \pi_i p_i^{-\alpha}\Big).$$

To complete the proof, note that, since $f(\theta)>0$, $w(\theta, M)>0$, $p(\theta, M)>0$, and $(M+s(\theta, M))>0$. Q.E.D.

Remark. In the absence of randomness (i.e. $n=1$), monetary policy remains effective. As we shall see below, however, active policy is not desirable in this case.

The effectiveness of monetary policy having been demonstrated, a choice criterion must be adopted for the selection of an optimal policy. We adopt an expected utility criterion. The function $l(\theta)$ is chosen so as to

$$\text{(8)} \qquad \max_{l_1,\ldots,l_n} \sum_k \pi_k\Big\{-l_k+\frac{1}{\alpha}(l_k\theta_k)^{\alpha}\Big\}$$

$$\text{s.t.} \quad l_k \geq 0, \qquad k=1,\ldots,n.$$

The solution to the maximization problem (8) is given by

$$\text{(9)} \qquad l_k^* = (\theta_k)^{\alpha/(1-\alpha)}, \qquad k=1,\ldots,n.$$

Remark. The choice of an expected utility criterion is crucial to the argument. Alternatively, one may consider a Pareto criterion, or, equivalently, the maximization of a weighted sum of the utility levels attained by agents of various types; i.e.

$$\sum_k \lambda_k\Big\{-l_k+\frac{1}{\alpha}\pi_j(l_j\theta_j)^{\alpha}\Big\}.$$

where $\lambda_k>0$, $\sum_k \lambda_k=1$. The optimal labour supply is then given by

$$\bar{l}_i=\Big(\frac{\pi_i}{\lambda_i}\Big)^{1/(1-\alpha)}\theta_i^{\alpha/(1-\alpha)}.$$

Observe that in the absence of active monetary policy we do indeed obtain a Pareto optimum with

$$\lambda_k = (\pi_k\theta_k^\alpha)/(\sum \pi_i\theta_i^\alpha).$$

This may be surprising, given the incompleteness of markets; it is due to the absence of the intra-generational heterogeneity.

Optimality as defined above does not necessarily imply stabilization of the level of output. A straightforward computation yields that the optimal rate of increase of the money stock is given by $\sigma_k^* - 1$, where

$$(10)\qquad \sigma_k^* = \frac{M + s_k^*}{M} = \theta_k^{-\alpha/(1-\alpha)}\Big(\sum_i \pi_i\theta_i^{\alpha/(1-\alpha)}\Big) = \frac{c}{l_k^*}, \qquad k = 1, \ldots, n$$

and the optimal expected rate of increase of the money stock is given by $\sigma^* - 1$, where

$$(11)\qquad \sigma^* = \sum \pi_k\sigma_k^* = \Big(\sum_k \pi_k\theta_k^{-\alpha/(1-\alpha)}\Big)\Big(\sum_k \pi_k\theta_k^{\alpha/(1-\alpha)}\Big)$$

Consequently, the optimal expected rate of increase of the money stock is greater than 0. Even though this result is dependent on the particular class of utility and production functions that we consider, it is independent of the particular value of α and of the distribution of the productivity parameter θ. In particular, it is independent of the stabilizing or destabilizing effects of optimal policy.

Suppose now that an optimal policy is followed, and imagine that an econometrician is trying to estimate the effectiveness of monetary policy. If he estimates

$$\hat{y}_t = \beta_0 + \beta_1\hat{\theta}_t + \beta_2\hat{\sigma}_{t-1}$$

where $\sigma_{t-1} = (M_t/M_{t-1})$ and the caret denotes the natural logarithm of the corresponding variable, he will find that the previous period's rate of money growth, $\hat{\sigma}_{t-1}$, has no effect on current output. If he is more perceptive, he will estimate the equation

$$\hat{y}_t = \beta_0 + \beta_1\hat{\theta}_t + \beta_2\hat{\sigma}_t$$

realizing that there is a lead in monetary policy, and that the effect of printing money is retroactive. Since an optimal policy is assumed to be followed, however, and since, for simplicity, the expression

$$c = \Big[\sum_k \pi_k\theta_k^{\alpha/(1-\alpha)}\Big]$$

can be assumed to equal 1, then

$$\hat{\sigma}_t = \frac{-\alpha}{1-\alpha}\hat{\theta}_t;$$

$\hat{\sigma}$ and $\hat{\theta}$ are collinear. Consequently, even though the two variables, θ and σ, combined completely determine output, the multicollinearity prevents an empirical determination of which of the two variables actually determines output.

To make the results more interesting, suppose that the monetary policy followed contained a random element: $\hat{\sigma}_t = \hat{\sigma}_t^+ + \varepsilon_t$ where ε is a variable distributed identically and independently across time with mean zero and variance $\sigma^2\varepsilon$. Given the distribution of ε, $\hat{\sigma}^+(\theta_t)$ can be chosen such that the same amount of labour is worked as when $\hat{\sigma}^*(\theta_t)$ is chosen in the certainty case.[5] That is, workers must decide how much to work based on their expectations of the lump-sum distributions. Previously we chose the distribution of $\hat{\sigma}(\theta_t)$ to be a single point $\hat{\sigma}^*(\theta_t)$, but it is also possible, given a distribution around a mean, to choose a mean for $\hat{\sigma}(\theta_t)$, called $\hat{\sigma}_t^+$, which leads the worker to choose to work the same amount as when he knows $\hat{\sigma}_t^*$ will be distributed. Since workers must decide how much labour to supply prior to the realization of the random variable ε, and since the distribution of this error term is starionary, the level of output is still directly related to the mean of monetary policy—in this case $\hat{\sigma}_t^+$. Furthermore, $\hat{y}_t = -(1/\alpha)\hat{\sigma}_t^*$ as before, because the randomness was injected in such a way that labour and output are the same as in the certainty case. Now suppose the econometrician tries to estimate the effect of monetary policy on current output, but fails to separate anticipated and unanticipated money growth. For a consistent policy, anticipated money is related one to one to the productivity level θ, which remains perfectly correlated with the labour supply and output:

$$\hat{y}_t = \frac{1}{1-\alpha}\hat{\theta}_t.$$

Since monetary growth $\hat{\theta}_t$ equals anticipated growth $\hat{\sigma}_t^+$ plus anticipated growth ε_t, there is no longer a perfect correlation between output and monetary growth. In estimating the equation

$$\hat{y}_t = \beta_0 + \beta_1\hat{\theta}_t + \beta_2\hat{\sigma}_t$$

where $\hat{\sigma}_t = \hat{\sigma}_t^+ + \varepsilon_t$, the econometrician has an errors-in-variables problem (because *he* should be separating anticipated and unanticipated money); in this special case the bias is such that the coefficient on money will be 0. (This is not an asymptotic result because of the perfect correlation of $\hat{y}_t$ and $\hat{\theta}_t$). *The Phillips curve (of output and future money growth) is mistakenly estimated to be vertical, even though monetary policy precisely determines output.*

II. Effective but not Omnipotent Policy

We now consider a situation where the government can influence but not absolutely control the level of employment and output. Specifically, we assume that monetary policy must be chosen with a lead. In terms of the model of the previous section, there are two equivalent ways of thinking of this assumption. One is that the government needs time to gear up its policy and so it must choose (M_{t+1}/M_t) before observing θ_t. Equivalently, the government can choose its policy for period $(t+1)$ after observing θ_t, but the lump sum it passes out must go to the young people of the following generation.

Consider a model of an economy otherwise identical to that described in Section I, except for the alternatives at the disposal of the monetary authority. The government chooses (possibly negative) lump-sum transfers, s, to be distributed in the following period to what is the young generation of the current period. Unlike the situation in the previous model, however, the government does *not* have a chance to learn the actual current value of the productivity parameter, θ, before it has to decide on the level of the lump-sum transfers.

A *state of the economy* is now an ordered triple $(\bar{\theta}, \theta, M)$, where $\bar{\theta}$ deontes the productivity level during the previous period, θ the productivity level in the current period, and M the current nominal money stock. Since the government does not have immediate knowledge of the parameter θ, a *monetary policy* is a function of the form $s(\bar{\theta}, M)$ such that $\{s(\bar{\theta}, M)+M\}>0$. It can be written as a vector $\mathbf{s}=(s_1, \ldots, s_n)$. A *price system* is a pair of functions $p(\bar{\theta}, \theta, M)$, $w(\bar{\theta}, \theta, M)$. It can be denoted by a pair of vectors $[\mathbf{p}, \mathbf{w}]=[(p_{1,1}, \ldots, p_{i,j}, \ldots, p_{n,n}), (w_{1,1}, \ldots, w_{i,j}, \ldots, w_{n,n})]$ where (i, j) denotes the state of the economy $(\bar{\theta}_i, \theta_j)$, which occurs with probability $\pi_{i,j}=\pi_i\pi_j$.

Given a monetary policy, $\mathbf{s}$, and a price system $[\mathbf{p}, \mathbf{w}]$, a young person in state (i, j) finds it optimal to supply labour according to

$$l_{i,j}(\mathbf{s}, \mathbf{p}, \mathbf{w})=\frac{1}{w_{i,j}}\left[\left\{w_{i,j}\left(\sum_k \pi_{j,k}p_{j,k}^{-\alpha}\right)\right\}^{1/(1-\alpha)}\left(\frac{M}{M+s_i}\right)^{\alpha/(1-\alpha)}-s_i\right] \tag{12}$$

$$i=1, \ldots, n; \quad j=1, \ldots, n.$$

An equilibrium for the economy is defined as follows.

Definition 2. A monetary policy $s(\bar{\theta}, M)$ and a price system $[p(\bar{\theta}, \theta, M), w(\bar{\theta}, \theta, M)]$ are an equilibrium for the economy if and only if

$$w_{i,j}l_{i,j}=M, \qquad i=1, \ldots, n; j=1, \ldots, n \tag{13a}$$

$$w_{i,j}=\theta_j p_{i,j}, \qquad i=1, \ldots, n; j=1, \ldots, n. \tag{13b}$$

The system of equations (13) is equivalent to the system

$$M+s_i=M^\alpha\theta_j p_{i,j}\left(\sum_k \pi_{j,k}p_{j,k}^{-\alpha}\right), \qquad i=1, \ldots, n; \quad j=1, \ldots m \tag{14a}$$

$$w_{i,j}=\theta_j p_{i,j} \tag{14b}$$

from which the following proposition follows immediately.

Proposition 3. *Given a monetary policy $s(\bar{\theta}, M)$, there exists a unique price system $[p(\bar{\theta}, \theta, M), w(\bar{\theta}, \theta, M)]$ such that the triple $[s(\bar{\theta}, M), p(\bar{\theta}, \theta, M), w(\bar{\theta}, \theta, M)]$ is an equilibrium for the economy.*

Proof. The argument follows closely that for Proposition 1. Let

$$p_{i,j}=(M+s_i)\frac{b_j}{\theta_j}, \tag{15}$$

$$w_{i,j}=(M+s_i)b_j$$

and

$$l_{i,j}=\frac{M}{M+s_i}\frac{1}{b_j}$$

where

$$b_j = \left(\frac{M+s_j}{M}\right)^{\alpha} b, \quad \text{with} \quad b = \left\{\sum_k \pi_k \theta_k^{\alpha}\left(\frac{M}{M+s_i}\right)^{\alpha^2}\right\}^{1/(\alpha-1)}$$

The system of equations (13) in Definition 2 is then clearly satisfied. Q.E.D.

We use the notation $\sigma(\bar{\theta}, M)$ for the ratio $\{s(\bar{\theta}, M)+M\}/M$.

In the context of this economy, all real variables are determined if we specify the labour supply function $f(\bar{\theta}, \theta)$—the labour supplied by the young generation as a function of the state of the economy. The question of effectiveness of monetary policy reduces to the question of whether different equilibrium labour supply functions can result from different monetary policy functions. The following argument demonstrates that, unlike the situation where monetary policy could adjust instantaneously to the particular disturbance, in the case of a lagged responce monetary policy is not omnipotent. From (15),

$$(16) \qquad l_{i,j} = (\sigma_i \sigma_j^{\alpha} b)^{-1}$$

and consequently,

$$(17) \qquad l_{i,i} = \sigma_i^{-(\alpha+1)} b^{-1}.$$

Equation (17) defines the trade-off faced by the monetary authority. Since $(l_{i,k}/l_{j,k}) = (\sigma_j/\sigma_i)$, and since furthermore $l_{i,j}$ is a function homogeneous of degree $\{1/(\alpha-1)\}$ in $(\sigma_1, \ldots, \sigma_n)$, monetary policy can be used to attain any arbitrary positive labour supply level for states of the economy of the form $(i, \bar{k})$, where $\bar{k}$ is a fixed index. The employment level in the remaining states is then uniquely determined. Observe that, even though monetary policy, owing to the lag, is not omnipotent anymore, it can still affect the equilibrium allocation.

The effectiveness of policy having been demonstrated, a choice must be made of the particular policy to be considered. If we suppose that the parameter α of the utility function is negative and large in absolute value, then an obvious candidate is the policy defined by

$$(18) \qquad l_{i,j}^{**} = \theta_i^{\alpha/(1-\alpha)(1+\alpha)} \theta_j^{\alpha^2/(1-\alpha)(1+\alpha)}.$$

For state of nature of the form (i, i) the policy is equivalent to the first-best policy derived under the no-lag assumption in the previous section ($l_i^* = \theta_i^{\alpha/(1-\alpha)}$). For states of nature of the form (i, j) with $i \neq j$, the policy is asymptotically equivalent to the first-best policy, since

$$\lim_{\alpha \to -\infty} (\theta_i^{\alpha/(1-\alpha)(1+\alpha)} \theta_j^{\alpha^2/(1-\alpha)(1+\alpha)}) = \theta_j^{-1}.$$

The monetary policy that implements the labour supply rule given by (18) can be computed as follows. From (17),

$$\sigma_k^{**} = \theta_k^{-\alpha/(1-\alpha)(1+\alpha)} b^{-1/(1+\alpha)}.$$

Substituting from (15) for b and solving, we obtain

$$(19) \qquad \sigma_{kk}^{**} = \theta_k^{-\alpha/(1-\alpha)(1+\alpha)} \sum_i \pi_i \theta_i^{\alpha/(1-\alpha)(1+\alpha)}.$$

Remark. Since the policy defined by (18) is only asymptotically optimal, it is of interest to look at the asymptotic form of the policy. From (19), it follows immediately that

$$\lim_{\alpha \to -\infty} \sigma_k^{**} = \sigma^{**} = 1.$$

That is, the asymptotically optimal policy is a k% rule. It is instructive that, even though the optimal policy is indeed—at the limit—a k% rule, the optimal rate of change of the money supply is determinate and equal to 0. Unlike the informationally based arguments, the resulting allocation is not invariant to the changes in the (constant) rate of change of the money supply.

Observe now that, if the government follows the asymptotically optimal policy (19), *output is serially correlated, even though the real disturbance (the labour productivity level, θ) is distributed independently across time.* When productivity is low, the monetary authority promises to print money in the future ($\theta_k > \theta_{k'} \Rightarrow \sigma_k^{**} > \sigma_{k'}^{*}$ for $\alpha \ll 0$). The point of such a policy is to increase current labour supply ($\partial l^{**}/\partial\theta = \alpha^2 c/(1-\alpha^2) < 0$) at the cost of reducing the expected labour supply of the following period ($\partial l^{**}/\partial\bar{\theta} = \alpha c/(1-\alpha^2) > 0$). *An increase in the money stock leads to increased short-run output but decreased future output, and causes a business cycle with output serially correlated.*

Finally, even though policy may be useful in the sense of increasing expected utility, in certain periods (states of the economy) the utility level attained is lower than in the no-policy case. To see this, suppose that

$$\sum_k \pi_k \theta_k^{\alpha} = 1, \qquad \theta_j = 1, \quad \theta_i \neq 1.$$

Then, according to the policy suggested above (and if $\alpha \ll 0$), $l_{i,j}^{**} \neq 1$, while under no policy does $l = 1$ for all states of nature, which is "optimal" for the state (i, j). The problem of sometimes running into a series of shocks that, in the short run, makes the economy worse off than it would have been in the no-policy case is the price of following an overall effective, but not omnipotent, policy.

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NOTES

[1] The model is an extension of that in Samuelson (1958).

[2] The linearity of the utility function with respect to labour is not important. One can use $u(l, c) = -(1/\beta)l^{\beta} + (1/\alpha)c^{\alpha}$ and derive the same conclusions, only with more complicated algebra.

[3] The assumption of independence is not important. It is made so as to simplify definitions and computations.

[4] Instead of a lump sum, we could have postulated the monetary policy to be a general linear function of the money holdings of young agents. By the rationality assumption on agents' expectations, however, the multiplicative term would be neutral; no loss of generality is involved when we limit ourselves to lump-sum mechanisms.

[5] The certainty equivalence assumption is, strictly speaking, false. It can be justified by interpreting equation (4) to be a first-order approximation of the "true" labour supply function. Since, on the other hand, no assumption has been made on the distribution of the random term, ε other than stationarity, it may be the case that it is such as to preserve the labour supply function.

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