EQUITY, EFFICIENCY, AND ADVANTAGEOUS RANDOMNESS

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I. INTRODUCTION

The argument that seeks to establish the existence of a tradeoff between equity and efficiency relies on the consideration of problems of incentives. It is derived from the Pareto optimality of a competitive general equilibrium, which, in turn, depends on the assumption of universality of markets. In situations involving uncertainty, the universality of markets requires that all economic agents be perfectly informed about the probability of occurrence of various contingencies. I shall look at the dependence between equity and efficiency in a context where the latter assumption fails—the system of markets is incomplete—and I shall demonstrate that, under plausible conditions, *equity promotes efficiency.* The intuition is clear: Equity economizes on the information necessary to attain an efficient allocation.

Efficiency in the allocation of wealth requires that individuals be able to transfer revenue across different states of nature. In the presence of uncertainty about future earnings, however, if the individual himself is the only one who knows the "true" distribution of this random variable, the problem of *adverse selection* arises.[1] An individual whose future income will be low with high probability has an incentive to purchase the insurance policy designed for another individual whose future earnings are likely to be high. If the two individuals are indistinguishable during the contracting period (i.e., no differential premium is possible), and if the payment to be made can be a function only of realized income, the second individual will not be able to allocate his wealth in an efficient manner. Consequently, if he is risk averse, he may find it preferable to have an income stream with lower mean, but with the possibility of insuring against any variance.

I shall construct a model to demonstrate that the situation described above can indeed occur: *In the presence of adverse selection, income transfers may be desirable to all, even when extended to the point of eliminating any differences in the distribution of income.*

The term income redistribution is employed here in a specific

1. See Arrow [1963].

The Quarterly Journal of Economics, August 1979 0033-5533/79/0093-0465$01.00

sense. Given a population divided into groups with different expected future incomes, differences among individuals can be eliminated by postulating that economic activity occurs before the agents know which group they belong to. Equivalently, the government can announce that, after all contracts have been negotiated, a lottery will reassign individuals to groups.

Green [1976] and Hirshleifer [1971] have analyzed situations where the social value information diverges from its private value. Furthermore, they have constructed examples where individuals would be willing to pay a positive price for information not to be revealed. Equivalently, the "average" member of the economy would be better off if information were not to be revealed. The argument developed in the present paper is stronger: I argue that *all* groups in the population can attain a higher level of expected utility if economic activity is to take place before information is revealed. Equivalently, *information may be not only ex ante undesirable, as Hirshleifer has argued, but ex post as well.*

Aumann and Peleg [1974], following Gale [1974], have constructed an example of a pure exchange economy where the competitive equilibrium after a trader has discarded some of his endowment is preferable for him to the competitive equilibrium that would have prevailed originally. There are two major differences between the Aumann-Peleg-Gale type of example and the argument presented here. First, since they work a complete Arrow-Debreu framework, they cannot hope for Pareto dominance. Second, their example involves no information-theoretic considerations that are the essence of mine.

Finally, Stiglitz [1975] has discussed the optimality properties of equilibria with different levels of screening. He postulates, however, that productivity is a decreasing function of variance, or heterogeneity, in the labor force and hence *cannot* raise the question dealt with in the present paper.

II. The Model and Results

Consider an economy lasting two periods and consisting of one unit of aggregate population. Period 1 is the contracting period, while an individual's second period income is a random variable $\tilde{w}$. The population is divided into two groups, a and b, of size λ_a and λ_b, respectively ($\lambda_a + \lambda_b = 1$). The random variable $\tilde{w}$ takes on two possible values, w^h and w^l, where $w^h > w^l > 0$. I shall denote by π the probability of low income (w^l), and by $1 - \pi$ the probability of high income

(w^h). The two groups differ in the value of π. Group a is the low-income group, while group b is the high-income group; i.e., $1 > \pi_a > \pi_b > 0$. An individual himself is the only one who knows which of the two groups he belongs to. The von Neumann-Morgenstern utility of income function is, for all individuals, of the form,

$$(1) \qquad u(w) = -e^{-\sigma w}, \quad \sigma > 0.$$

Consequently, the index of absolute risk aversion is independent of the level of income w and equal to σ.

An insurance contract available at period 1 specifies a premium p to be paid regardless of the income realized, and a payment x to be received by the individual in case his income turns out to be low. I shall assume that if a contract (p,x) is available, a consumer can buy as many of these contracts as he desires. If he buys k $(k \geq 0)$ contracts, he pays a premium kp, and if his income turns out to be low, he receives a payment of kx. After normalization, it can be assumed that the premium p is equal to 1, and the contract is completely specified by the value of x. Given a contract x, if an individual buys k contracts, his expected utility can be computed as follows:

$$(2) \qquad u(k,x) = -\pi e^{-\sigma(w^l + k(x-1))} - (1-\pi)e^{-\sigma(w^h - k)}.$$

Consequently,

$$(3) \qquad \frac{\partial u}{\partial k}(k,x) = \pi\sigma(x-1)e^{-\sigma(w^l + k(x-1))} - (1-\pi)\sigma e^{-\sigma(w^h-k)},$$

and

$$(4) \qquad \frac{\partial^2 u}{\partial k^2}(k,x) = -\pi\sigma^2(x-1)^2 e^{-\sigma(w^l+k(x-1))} - (1-\pi)\sigma^2 e^{-\sigma(w^h-k)}.$$

Since, for all k, $(\partial^2 u/\partial k^2)(k,x) < 0$, the optimal number of contracts x purchased by an individual with probability of low income equal to π and risk aversion equal to σ is given by

$$(5) \qquad k(x;\pi,\sigma) = \max\left\{0, \frac{1}{\sigma x}\left(\log\frac{\pi(x-1)}{(1-\pi)} + \sigma(w^h - w^l)\right)\right\}.$$

Observe that given π and $x > 1$, there exists $\bar{\sigma}(x,\pi)$, such that if $\sigma > \bar{\sigma}(x,\pi)$, $k(x;\pi,\sigma) > 0$.

If the two income groups were identifiable, two insurance contracts would be offered—x_a and x_b—each sold to the members of only one group. Setting $x_a = 1/\pi_a$ and $x_b = 1/\pi_b$ would yield a first-best allocation of income for each group. Members of groups a and b would buy $k_a = \pi_a(w^h - w^l)$ and $k_b = \pi_b(w^h - w^l)$ contracts, respectively. Incomes would be equal to $\pi_a w^l + (1 - \pi_a)w^h$ and $\pi_b w^l + (1 - \pi_b)w^h$ independently of the state of nature, the levels of utility attained would equal

$$u_a^* = -e^{-\sigma(\pi_a w^l + (1-\pi_a)w^h)}$$

and

$$u_b^* = -e^{-\sigma(\pi_b w^l + (1-\pi_b)w^h)},$$

and the expected profit of the insurance company would be 0.

The two groups are *not*, however, identifiable, and members of the low-income group a have an incentive to purchase the contract designed for the high-income group b, x_b. Clearly, not more than one contract can survive in the market.

The expected profit of the insurance company from offering the contract x is given by

$$R(x) = \lambda_a k_a(1 - \pi_a x) + \lambda_b k_b(1 - \pi_b x), \tag{6}$$

where k_i is an abbreviation for $k(x;\pi_i,\sigma)$, $i = a,b$. I shall say a contract is *feasible* if and only if $R(x) \geq 0$. A contract with $x > 1/\pi_b$ is clearly infeasible. Furthermore, since I am interested in the maximum utility attained by groups a and b subject to the condition of feasibility, I shall assume that $x > 1/\pi_a$; i.e., $x \epsilon (1/\pi_a, 1/\pi_b)$.

The level of expected utility attained by group i ($i = a,b$) if contract x is offered in the market equals

$$\hat{u}_i = -\pi_i e^{-\sigma((1-1/x)w^h + (1/x)w^l) + 1/x \log Q_i}[\pi_i e^{-\log Q_i} + 1 - \pi_i], \tag{7}$$

where

$$Q_i = \frac{\pi_i(1 - 1/x)}{(1 - \pi_i)1/x}, \quad i = a,b.$$

In computing (7), I have assumed that $k_i > 0$ for both $i = a$ and $i = b$. This is correct for σ high, and since the argument to follow will depend on having σ higher than some minimum level, the assumption is justified.

Let us now consider what an egalitarian version of the economy looks like. The government announces that after individuals have contracted with the insurance company, a lottery is to reallocate individuals between the two groups a and b, keeping their relative sizes the same. Equivalently, all income over w^l is to be taxed at 100 percent and reallocated randomly in the population. Formally, the economy then consists of one aggregate unit of population whose second period income is w^l with probability $\overline{\pi}$ and w^h with probability $(1 - \overline{\pi})$, where

$$\overline{\pi} = \lambda_a \pi_a + \lambda_b \pi_b. \tag{8}$$

The population is now homogeneous, and the insurance company can offer the contract $x = 1/\overline{\pi}$, which yields a first-best allocation of wealth. The level of expected utility attained by an individual in the egalitarian economy equals

$$\overline{u} = -e^{-\sigma((1-\overline{\pi})w^h + (\overline{\pi})w^l)}. \tag{9}$$

I shall now show that if the individuals in the economy are strongly risk-averse, $\overline{u}$ is higher than $\hat{u}_a$ as well as $\hat{u}_b$.

Since $\overline{u}, \hat{u}_a, \hat{u}_b$ are all negative numbers, $\hat{u}_a < \hat{u}$ and $\hat{u}_b < \overline{u}$ if and only if $\hat{u}_a/\overline{u} > 1$ and $\hat{u}_b/\overline{u} > 1$. These ratios can be computed as follows:

$$\hat{u}_a/\overline{u} = e^{-\sigma(\overline{\pi}-1/x)(w^h-w^l)} e^{(1/x)\log Q_a} [\pi_a e^{-\log Q_a} + 1 - \pi_a]; \tag{10}$$

$$\hat{u}_b/\overline{u} = e^{-\sigma(\overline{\pi}-1/x)(w^h-w^l)} e^{(1/x)\log Q_b} [\pi_b e^{-\log Q_b} + 1 - \pi_b]. \tag{11}$$

Since the contract x is feasible, $R(x) \geq 0$. Since $\pi_a > \pi_b$, $(1 - \pi_a x) < (1 - \pi_b x)$ and $k_a > k_b$. Furthermore, $(1 - \pi_a x) > 0$. Consequently, $\lambda_a k_a(1 - \pi_a x) + \lambda_b k_a(1 - \pi_b x) > \lambda_a k_a(1 - \pi_a x) + \lambda_b k_b(1 - \pi_b x) = R(x) \geq 0$. Since $k_a > 0$, $1 - \overline{\pi}x > 0$, or $(\overline{\pi} - 1/x) < 0$. Consequently the exponent $-\sigma(\overline{\pi} - 1/x)(w^h - w^l)$ is positive and increasing with σ. For σ large, $u_a/\overline{u} > 1$, and $u_b/\overline{u} > 1$. We have demonstrated that *there exists a $\overline{\sigma} > 0$ such that, if $\sigma > \overline{\sigma}$, the low- as well as the high-income groups are better off in the egalitarian economy.*[2]

That the low-income group gains from the equalization of the income distribution is clear. That the high-income group gains as well can be understood as follows: The loss in expected income is compensated for by the ability to obtain an efficient allocation across states of nature.

III. Conclusion

The model presented above lacked generality in a number of respects, but the simplifications were innocuous. Individuals were

2. The argument can be made that to compare the two alternatives one must also take into account the profits made by the insurance firm. This is perfectly consistent with the present analysis if one restricts his attention to the contract such that $R(x) = 0$.

endowed with a constant absolute risk aversion utility function so that the importance of attitudes toward risk could be demonstrated clearly. Alternatively, a general utility function could have been used, and a Taylor approximation argument employed to yield the same conclusions. That the future income of individuals could take on only two possible values is inherent to the insurance problem. The model could be easily extended to allow for a continuous distribution of π as well as for variations in the risk aversion parameter σ among individuals.[3]

The intuition behind the result is clear: The low-income group creates an externality that prevents the high-income group from attaining a first-best allocation of its wealth. In the egalitarian economy the externality vanishes, and the increase in the efficiency of allocation may outweigh the loss in income by the high-income group.

In general, the more homogeneous a population, the lower the amount of information required to attain a first-best allocation. If the cost of misallocation of wealth is high, the elimination of all differences among agents becomes a preferred alternative.

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3. For the result to follow in the extended model, it must be assumed that the elasticity $\sigma'(\pi)/\sigma(\pi)$ is low. Anyway, there is no good reason why π and σ should be related at all.